

12-13-2019

Multi-sensor Data Fusion with Network Delays and Correlated Noises

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Recommended Citation

Zhang Shankai, Zhu Cui, Su Zhong, Dai Juan. Multi-sensor Data Fusion with Network Delays and Correlated Noises[J]. Journal of System Simulation, 2019, 31(12): 2617-2625.

Multi-sensor Data Fusion with Network Delays and Correlated Noises

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Abstract: This paper focuses on the state estimation for multi-sensor system with network delays and correlated noises. An orthogonal transformation method is applied to remove the correlations between different noises. For the problem of packet delays due to the unreliable network, a buffer with certain length is introduced to store the measurements, and the measurements are reordered using a timestamp in the buffer. *Based on that, a novel sequential data fusion algorithm is proposed, by which the influence of the noise correlation and packet delays can be weakened effectively.* Compared with the traditional sequential fusion method, the proposed algorithm has higher estimation accuracy. Simulation results show the effectiveness of the proposed algorithms.

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考虑相关噪声与网络延迟的多传感器数据融合

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摘要: 针对多传感器系统中存在的相关噪声以及网络延迟问题, 提出了一种带有缓存器的序贯式融合滤波算法。利用正交迭代法解除了噪声之间的相关性。针对不可靠网络导致的延迟甚至丢包问题, 引入缓存器存储量测值, 并在缓存器中利用时间戳对量测值进行重新排序。在此基础上提出了一种新型低维序贯式融合滤波算法, 相比于传统的序贯融合算法具有更好的估计性能, 同时能有效降低噪声相关和网络延时对系统性能造成的影响。最后仿真验证了该算法的有效性。

关键词: 相关噪声; 通信延迟; 正交变换; 缓存器; 序贯式融合滤波器

中图分类号: TP274+.2 文献标识码: A 文章编号: 1004-731X (2019) 12-2617-09

DOI: 10.16182/j.issn1004731x.joss.19-FZ0354E

Introduction

Wireless multi-sensor network has attracted significant attention in recent years. Comparing with

single sensor system, multi-sensor system can get higher accuracy, and the complexity of wiring can be reduced by wireless network. Therefore, wireless network system has been successfully applied in an extensive range of areas. Meanwhile, the wireless multi-sensor system also has some shortcomings, such as measurement-delay or loss and the correlation between channels. Therefore, many



Received: 2019-05-30 Revised: 2019-07-20;
Foundation: National Natural Science Foundation of China (61603047), Scientific Research Project of Beijing Municipal Educational Commission (KM201911232014);
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research results have been proposed for these problems^[1].

Multi-sensor system has more channels than single sensor system, so the crosstalk between channels is inevitable. A fusion algorithm with multiple channels and single channel is considered in Ref.[2]. The paper proposes the best linear unbiased estimation under minimization of the mean square error, and proves that it is always convergent. In particular, in Ref.[3], an optimal unbiased finite impulse response algorithm which consists of a robust UFIR filter implemented by a smart sensor is studied. The algorithm is more robust to noise statistical errors and model uncertainties. Ref.[4] develops the asynchronous correlation between system and measurements in the non-linear multi-sensor multi-rate system. Based on Ref. Ref.[3], a distributed fusion algorithm for the correlation between measurements noises is presented in Ref.[5]. In the other hand, an optimal distributed robust Kalman-type recursive filter with auto-correlated and cross-correlated noises is proposed in Ref.[6]. Recently, Caballero-Águila presents a distributed filter in which the process and measurement noises are one-step auto-correlated and two-step cross-correlated in Ref.[7]. Meanwhile, Ref.[8] proposes a model not only with correlated noises, but also with unreliable measurements which is common in wireless network system.

From Ref.[8], we can see that the measurement-delays are the focus of recent research results too. A distributed estimation fusion algorithm for multi-rate multi-sensor systems with measurement delays is presented in Ref.[9]. The boundary of delay steps due to network topology and agent dynamics is given in Ref.[10] by employing algebraic Riccati equation.

Yanyan Hu and Zengwang Jin concern the fusion estimation problem for stochastic systems with asynchronous multi-sensors and multiple packet dropouts in Ref.[11]. The asynchronous sensor measurements will be aligned to the fusion time before fusing the estimates. In Ref.[12], a finite length buffer is used to deal with measurement delay or loss, and a distributed federated Kalman filter fusion is presented. An optimal asynchronous estimation fusion algorithm is derived in Ref.[13] based on the transformed equivalent measurement. The algorithm in this paper can have arbitrary sampling rates and arbitrary initial sampling instants. Meanwhile, an optimal distributed fusion method for the system with delay and uncertain observation is given in Ref.[14]. For stability of multi-channel decentralized systems, Ref.[15] gives the upper and lower bound conditions that the packet loss rate should be satisfied when the system is stable. Taking into account the multi-rate problem, the state estimation problem of multi-rate multi-sensor systems with correlated noises is studied in Ref.[16]. Meanwhile, the influence of unreliable measurements and correlated noises is analyzed in Ref.[17], where the method of decorrelation is the same as the method in Ref.[16].

From the aforementioned analysis, we know that the noises correlation and measurement-delays which are both common in wireless multi-sensor network are rarely considered simultaneously in previous research. Focusing on that, this paper presents a sequential fusion center decorrelated through orthogonal transformation, and a buffer with certain length is proposed to store the measurements which are delayed. The performance is analyzed.

The rest of this paper is organized as follows: The stochastic system model with measurement-

delays and noise correlations is formulated in Section 2. The process of decorrelation, the storage method of the buffer and a new data fusion estimation method are proposed in Section 3. Section 4 gives the simulation results to illustrate the effectiveness of the proposed algorithm. Finally, Section 5 gives the conclusions.

Notation: Throughout this paper, $(\cdot)^T$ and $(\cdot)^{-1}$ denote the transpose and the inverse of the matrix, respectively. \mathbb{R}^n stands for the n -dimensional Euclidean space. $\mathbb{R}^{m \times n}$ defined as the set of all $m \times n$ real matrices. The statistical expectation is denoted as $E\{\cdot\}$. $E\{x|y\}$ is the expectation value of x under the condition y . The matrix \mathbf{I} and $\mathbf{0}$ are the identity matrix and the zero matrix of an appropriate size, respectively.

1 Problem Formulation

Consider the following discrete linear stochastic system:

$$x(k+1) = F(k)x(k) + w(k) \quad (1)$$

$$y_i(k) = H_i(k)x(k) + v_i(k) \quad i=1,2,\dots,N \quad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the system, $y_i(k) \in \mathbb{R}^{m_i}$ is the measurement vector of the sensor i , $F(k) \in \mathbb{R}^{n \times n}$ and $H_i(k) \in \mathbb{R}^{m_i \times n}$ are known matrices, $w(k)$ and $v_i(k)$ are zero-mean Gaussian white noises. The initial state $x(0)$ is independent of $w(k)$ and $v_i(k)$. It is assumed to be Gaussian distributed with $E\{x(0)\} = \mu_0$ and $E\{[x(0) - \mu_0][x(0) - \mu_0]^T\} = P_0$. $w(k)$ and $v_i(k)$ meet statistical properties as follows:

$$E\{w(k)\} = 0 \quad (3)$$

$$E\{v_i(k)\} = 0 \quad (4)$$

$$E\{w(k-1)v_i^T(l)\} = 0 \quad (5)$$

$$E\{w(k)w^T(l)\} = Q(k)\delta_{kl} \quad (6)$$

$$E\{v_i(k)v_j^T(l)\} = R_{ij}(k)\delta_{kl} \quad (7)$$

where δ_{kl} is the kronecker function. The process

noises are cross-correlated with the measurement noises one-step apart and the measurement noises are correlated between different sensors during a fusion interval which is expressed in Eq.(6) and Eq.(7). For convenience, $R_{ij}(k)$ is abbreviated as $R_i(k)$ when $i=j$.

The measurements of local sensors are time-stamped, and then transmitted through the wireless network to the fusion center directly. Moreover, the wireless transmission channel is unreliable, which means that the system may suffer measurement delays or losses during the transmission due to the unreliability. The measurements will reorder by the time-stamping when they arrive at the fusion center. For the model in this paper, each measurement delays d_k^i step, which satisfies the following random distribution:

$$f_i(m) = \Pr\{d_k^i = m\} \quad m=1,2,\dots, \quad i=1,2,3,\dots,N$$

It is assumed that d_{i1}^{k1} and d_{i2}^{k2} are independent with each other, if $i1 \neq i2$ or $k1 \neq k2$. Furthermore, d_k^i is also independent of $w(t)$, $v_i(t)$ and the initial state $x(0)$.

In this paper, all the data that arrive correctly at the fusion center will be stored in a buffer with length L ^[17]. If $y_i(t-L+h)$ ($h=1,2,\dots,L$) does not arrive between $t-l+h$ and t , it will be considered to be lost. This process is modeled by a random variable $\gamma_i^k(t)$:

$$\gamma_i^k(t) = \begin{cases} 1, & \text{if the measurements arrives at or before } t \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

As shown in (8), if $\gamma_i^k(t)=1$, then $\gamma_i^{k+l}(t)=1 \quad \forall l \in N$, that means if $y_i(k-L+h)$ ($h=1,2,\dots,L$) arrives at the fusion center at or before t , it will be considered to be present at all the time in the future. The storage method of the buffer at time t can be written as:

$$y_i^k(t) = \gamma_i^k(t)H_i(t)x(t) + \gamma_i^k(t)v_i(t)$$

Let $\tilde{v}_i(t) = \gamma_i^k(t)v_i(t)$, then

$$\tilde{R}_{ij}(t) = E\{\tilde{v}_i(t)\tilde{v}_j^T(t)\}$$

where $\tilde{R}_{ij}(k)$ is abbreviated as $\tilde{R}_i(k)$ if $i = j$, and

$$\tilde{R}_i(t) = \begin{cases} R_i(t), & \text{if the measurement reaches at or before } t, \\ \sigma^2 I, & \text{otherwise.} \end{cases} \quad (9)$$

From Eq.(9), we can know that the variance of the measurement noises at t is $R_i(t)$, if $\gamma_i^k(t)=1$, otherwise is $\sigma^2 I$, where $\sigma \rightarrow \infty$.

The optimal estimation problem is defined as follows:

$$\hat{x}(k, k) \triangleq E\{x(k) | Y(k), \gamma(k)\}$$

$$\hat{x}(k, k-1) \triangleq E\{x(k) | Y(k-1), \gamma(k-1)\}$$

$$Y(k) = (y^t(1), y^t(2), \dots, y^t(k))$$

$$Y(k) = (y^t(1), y^t(2), \dots, y^t(k))$$

$$\gamma(k) = (\gamma^t(1), \gamma^t(2), \dots, \gamma^t(k))$$

$$\gamma^t(k) = ((\gamma_1^t(k))^T, (\gamma_2^t(k))^T, \dots, (\gamma_N^t(k))^T)^T$$

2 Main result

2.1 Decorrelation

As shown in Eq.(6) and Eq.(7), there are two kinds of noise correlation have been considered in this paper. For simplicity, the cross-correlation between measurement noises and process noises is called the correlation Eq.(1) and the auto-correlation between the measurement noises of different sensors is called the correlation Eq.(2). We adopt an orthogonal transformation method to remove the noise correlations^[19].

Define an augmented state vector:

$$X(k) = \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}$$

then we have

$$X(k) = F^*(k-1)X(k) + w^*(k-1) \quad (10)$$

where

$$F^*(k) = \begin{bmatrix} F(k) & 0 \\ 0 & F(k-1) \end{bmatrix}$$

$$w^*(k-1) = [w^T(k-1) \quad w^T(k-1)]^T$$

The measurement equation can be rewritten as

$$y_i(k) = h_i(k)X(k) + v_i(k) \quad (11)$$

where

$$h_i(k) = [H_i(k) \quad 0]$$

and

$$E\{w^*(k-1)v_i^T(k)\} = \begin{bmatrix} S_i(k) \\ 0 \end{bmatrix} \quad (12)$$

The prediction for Eq. (10) is

$$\hat{X}(k, k-1) = \begin{bmatrix} \hat{x}(k, k-1) \\ \hat{x}(k-1, k-1) \end{bmatrix} \quad (13)$$

Define:

$$P^\times(k, k-1) = E\left\{ \begin{matrix} (X(k) - \hat{X}(k, k-1)) \\ (X(k) - \hat{X}(k, k-1))^T \end{matrix} \right\}$$

then

$$P^\times(k, k-1) = \begin{bmatrix} P(k, k-1) & c \\ c^T & P(k-1, k-1) \end{bmatrix} \quad (14)$$

where

$$c = E\{\tilde{x}(k, k-1)\tilde{x}(k, k-1)^T\} = F(k-1)P(k-1, k-1)$$

A. Remove correlation Eq.(1)

Firstly, introducing an intermediate variable $T_i(k) = S_i^T(k)Q^{-1}(k)$ and using Eq.(11), the measurement equation is rewritten as

$$y_i(k) = h_i(k)X(k) + v_i(k) + T_i(k)[x(k) - F(k-1)x(k-1) - w(k-1)] = \quad (15)$$

$$H^\times(k)X(k) + v_i^\times(k)$$

where

$$H_i^\times(k) = [H_i(k) \quad 0] + [T_i(k) \quad -T_i(k)F(k-1)] \quad (16)$$

$$v_i^\times(k) = v_i(k) - T_i(k)w(k-1)$$

From Eq.(15) and Eq.(16), we have

$$E\{w^*(k-1)(v_i^\times)^T\} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (17)$$

$$R_i^\times(k) = E\{v_i^\times(k)(v_i^\times(k))^T\} = R_i(k) - T_i(k)S_i(k) \quad (18)$$

$$R_{i,j}^\times(k) = E\{v_i^\times(k)v_j^\times(k)\} = R_{ij}(k) - T_i(k)S_j(k) \quad (19)$$

According to Eq.(17), we know that the

correlation Eq.(1) has been removed.

B. Remove correlation Eq.(2)

Although correlation Eq.(1) has been removed in 3.1-A, the correlation Eq.(2) still exists as shown in Eq.(19). Next, we will remove correlation Eq.(2).

Define

$$\begin{aligned} g_1^i(k) &= R_{i,1}^*(k)(R_1^*(k))^{-1} \\ g_2^i(k) &= R_{i,2}^*(k)(R_2^*(k))^{-1} \\ &\vdots \\ g_{i-1}^i(k) &= R_{i,i-1}^*(k)(R_{i-1}^*(k))^{-1} \end{aligned} \quad (20)$$

and let

$$v_i^*(k) = v_i(k) - \sum_{j=1}^{i-1} g_j^i(k)v_j^*(k)$$

where

$$\begin{aligned} R_{i,1}^*(k) &= E\{v_i(k)(v_1^*(k))^T\} = R_{i,1}(k) \\ R_{i,2}^*(k) &= E\{v_i(k)(v_2^*(k))^T\} = \\ &R_{i,2}(k) - R_{i,1}^*(k)(g_1^2(k))^T \\ &\vdots \\ R_{i,i-1}^*(k) &= E\{v_i(k)(v_{i-1}^*(k))^T\} = \\ &R_{i,i-1}(k) - \sum_{j=1}^{i-2} R_{i,j}^*(k)(g_j^{i-1}(k))^T \end{aligned}$$

then the measurement equation can be rewritten as following:

$$y_i(k) = H_i(k)x(k) + v_i(k) + G_i(k)(Z_{i-1}^*(k) - (\psi_{i-1}^*(k)X(k) + V_{i-1}^*(k))) \quad (21)$$

where

$$\begin{aligned} Z_i^*(k) &= [(y_1^*(k))^T, (y_2^*(k))^T, \dots, (y_i^*(k))^T]^T \\ \psi_i^*(k) &= [(H_1^*(k))^T, (H_2^*(k))^T, \dots, (H_i^*(k))^T]^T \\ V_i^*(k) &= [(v_1^*(k))^T, (v_2^*(k))^T, \dots, (v_i^*(k))^T]^T \\ G_i^*(k) &= [g_1^i(k), g_2^i(k), \dots, g_{i-1}^i(k)] \end{aligned}$$

Let

$$y_i^*(k) = y_i(k) - G_i(k)Z_{i-1}^*(k) \quad (22)$$

$$H_i^*(k) = H_i(k) - G_i(k)\psi_{i-1}^*(k) \quad (23)$$

$$v_i^*(k) = v_i(k) - G_i(k)v_{i-1}^*(k) \quad (24)$$

then we can get

$$y_i^*(k) = H_i^*(k)x(k) + v_i^*(k) \quad (25)$$

where $v_i^*(k)$ satisfies:

$$\begin{aligned} E\{w(k)(v_i^*(k))^T\} &= 0 \\ E\{v_1^*(k)(v_i^*(k))^T\} &= 0 \\ &\vdots \\ E\{v_{i-1}^*(k)(v_i^*(k))^T\} &= 0 \end{aligned}$$

As shown above, the process noises are no longer correlated with the measurement noises, and the correlations between measurement noises from different sensors are removed successfully too.

2.2 Determination of the update time

Now we will determinate the update time of the buffer.

Ref.[18] presents the method to compute the length of each channel L_i . The buffer length in this algorithm is $L = \max\{L_i\}$ i.e., the buffer length in this paper is the longest length of the single-channel buffer. $Z^k(t) = ((y_1^k(t))^T, (y_2^k(t))^T, \dots, (y_N^k(t))^T)^T$ ($t = k - L + 1, \dots, k$) are stored in the $t - L + k$ slot of the buffer at time k , the dummy variable $\mathbf{0}$ would be stored in the corresponding slot if the measurement has not been received until time k .

Define

$$\tau = \begin{cases} \min\left\{t \mid \sum_{i=1}^N \gamma_i^k(t) > \sum_{i=1}^N \gamma_i^{k-1}(t), k - L + 1 \leq t < k\right\} \\ k \quad \text{otherwise.} \end{cases} \quad (26)$$

From Eq.(26), it can be seen that τ is the earliest time, when at least one measurement $y_i^t(\tau)$ is received by the fusion center at t . Fig. 1 shows an example of τ .

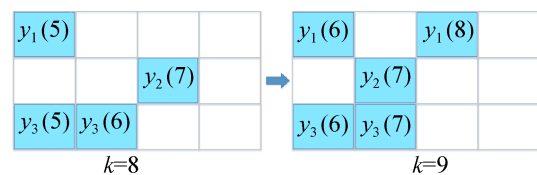


Fig. 1 Example of τ

In Fig. 1, the blue cell means that the corresponding measurement is stored in the buffer at current time, i.e., $y_1^8(5)$, $y_2^8(5)$, $y_1^8(7)$, $y_3^8(5)$, $y_3^8(6)$ arrive at or before time 8, $y_1^9(6)$ arrives at time 9, so the earliest update time is $\tau=6$.

Now we will give the sequential filtering method with communication delays and correlated noises.

Algorithm 1 The sequential filtering method with communication delays and correlated noises.

As shown in Eq.(26), τ is the earliest time, when at least one measurement $y^i(\tau)$ is received by the fusion center at t , so we can get

$$\hat{x}_N^k(\tau-1, \tau-1) = \hat{x}_N^{k-1}(\tau-1, \tau-1)$$

$$P_N^k(\tau-1, \tau-1) = P_N^{k-1}(\tau-1, \tau-1)$$

step 1: Initialization

$$\hat{X}_0^k(\tau, \tau) = \begin{bmatrix} F(\tau-1)\hat{x}_N^k(\tau-1, \tau-1) \\ \hat{x}_N^k(\tau-1, \tau-1) \end{bmatrix} \quad (27)$$

$$P_0^{k*}(\tau, \tau) =$$

$$\begin{bmatrix} F(\tau-1)P_N^k(\tau-1, \tau-1)F^T(\tau-1) + Q & F(\tau-1)P_N^k(\tau-1, \tau-1) \\ (F(\tau-1)P_N^k(\tau-1, \tau-1))^T & P_N^k(\tau-1, \tau-1) \end{bmatrix} \quad (28)$$

step 2: Measurement update

$$K_i^k(t) = P_{i-1}^{k*}(t, t) [H_i^*(t)]^T [H_i^*(t)P_{i-1}^{k*}(t, t) \times (H_i^*(t))^T + \bar{R}_i^*(t)]^{-1} \quad (29)$$

$$\hat{X}_i^k(t, t) = \hat{X}_{i-1}^k(t, t) + \gamma_i^k(t) K_i^k(t) [y_i^*(t) - H_i^*(t)\hat{X}_{i-1}^k(t, t)] \quad (30)$$

$$P_i^{k*}(t, t) = [I - \gamma_i^k(t) K_i^k(t) H_i^*(t)] P_{i-1}^{k*}(t, t) \quad (31)$$

$\hat{X}_N^t(k, k)$ and $P_N^{t*}(k, k)$ can be obtained by

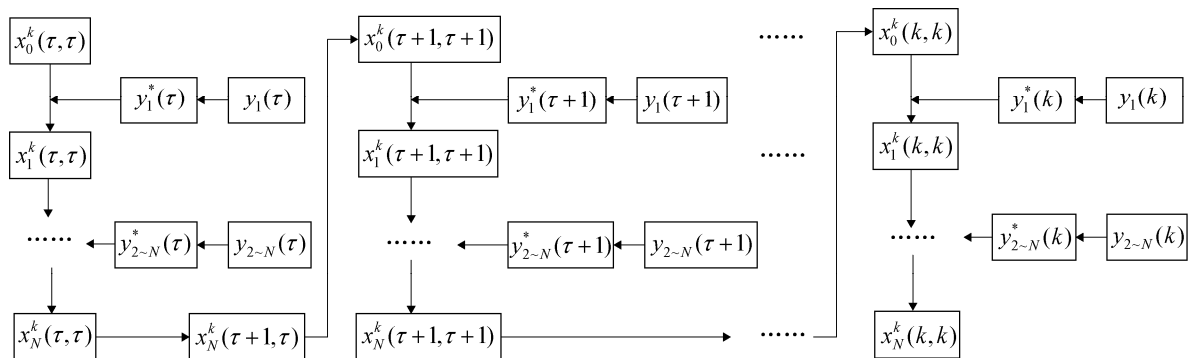


Fig. 2 Flow chart of the proposed algorithm

iterating N times

step 3: Extract the estimated value

$$\hat{x}^k(t, t) = \hat{x}_N^k(t, t) \quad (32)$$

$$P(t, t) = P_N^k(t, t) \quad (33)$$

step 4: Prediction

$$\hat{X}_0^k(t+1, t+1) = \begin{bmatrix} F(t)\hat{x}_N^k(t, t) \\ \hat{x}_N^k(t, t) \end{bmatrix} \quad (34)$$

$$P_0^{k*}(t+1, t+1) =$$

$$\begin{bmatrix} F(t)P_N^k(t, t)F^T(t) + Q & F(t)P_N^k(t, t) \\ (F(t)P_N^k(t, t))^T & P_N^k(t, t) \end{bmatrix} \quad (35)$$

step 5: Repeat Step 2 to Step 4 for $k - \tau$ times, we can get $\hat{x}^k(k, k)$ and $P(k, k)$.

step 6: Get the final result

$$\hat{x}(k, k) = \hat{x}^k(k, k)$$

The flow chart of the novel sequential algorithm

is given by Fig. 2.

Theorem 1: Consider the stochastic linear system given in Eq.(1) and Eq.(2), and the package arrival process Eq.(8). Let $\hat{x}(k, k)$ be the estimation computed by algorithm 1 with a certain length buffer, and $\bar{x}(k, k)$ is the estimation without buffer. Assume

$$P(k, k) = E\{(x(k) - \hat{x}(k, k))(x(k) - \hat{x}(k, k))^T\}$$

$$\bar{P}(k, k) = E\{(x(k) - \bar{x}(k, k))(x(k) - \bar{x}(k, k))^T\}$$

then

$$P(k, k) \leq \tilde{P}(k, k) \tag{36}$$

Proof: According to the definition of $\hat{x}(k, k)$, $\hat{x}(k, k) \triangleq E\{x(k)|Y(k), \gamma(k)\}$ where $Y(k) = (y^t(1), y^t(2), \dots, y^t(k))$, $y^t(k) = ((y_1^t(k))^T, (y_2^t(k))^T, \dots, (y_N^t(k))^T)^T$. We can get that $Y(k)$ include the measurements which arrive on time and delay no more than $L-1$ steps. Meanwhile, $\tilde{x}(k, k)$ is the estimation without buffer, so $\tilde{Y}(k)$ only includes the measurements arriving on time. Obviously, $Y(k)$ includes more messages than $\tilde{Y}(k)$. In other words, $Y(k)$ contains $\tilde{Y}(k)$, so we can get the conclusion that $P(k, k) \leq \tilde{P}(k, k)$.

3 Simulation

In this section, we present an example to verify the effectiveness of the algorithm. System parameters are set as following:

$$F(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad H_1 = [0.5 \quad 0],$$

$$H_2 = [0.5 \quad -0.5], \quad Q = \begin{bmatrix} 1/3 & 1/2 \\ 1/2 & 1 \end{bmatrix}$$

$v_i(k)$ is defined as following:

$$v_1(k) = A_1 * w(k) + \Theta_1(k)$$

$$v_2(k) = A_2 * w(k) + \Theta_2(k)$$

where: $A_1 = [4 \quad 8]$, $A_2 = [4.5 \quad 7.5]$. $\Theta_1(k)$ and $\Theta_2(k)$ are independent additive white gauss noises, and the covariance of $[\Theta_1(k), \Theta_2(k)]$ is

$$E \left\{ \begin{bmatrix} \Theta_1(k) & \Theta_2(k) \end{bmatrix}, \begin{bmatrix} \Theta_1^T(k) \\ \Theta_2^T(k) \end{bmatrix} \right\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We assume that the number of data delay step satisfied the Poisson distribution^[20] with an average value of d_i , that is, its probability density function satisfies:

$$f_i(m) = \frac{(d_i)^m e^{-d_i}}{m!}, m = 0, 1, \dots$$

where $d_i = E\{d_k^i\}$ represents the mean of the number of delay steps for each channel. In the simulation of this section, we assume $d_1 = 3$ and $d_2 = 2$.

The results of the simulation of two channels of the system state are shown in Fig. 3 and Fig. 4. This is to be expected, the traditional sequential filter (green line) can trace the real state (red line), but the algorithm in this paper (blue line) can trace better obviously, whatever in channel 1 or channel 2, so the algorithm in this paper has better performance than traditional fusion center. The traces of the error covariance of traditional sequential filter and the algorithm in this paper are shown in Fig. 5, from which we can see that the trace of error covariance for the algorithm in this paper is always lower than the traditional sequential filter. From this example, we can see that the algorithm in this paper can improve the estimation accuracy when the noises are correlated.

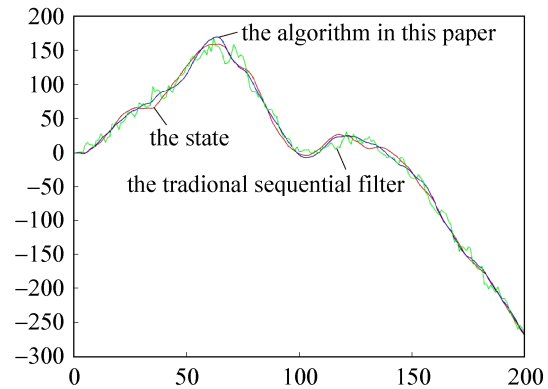


Fig. 3 Estimation curves of channel 1

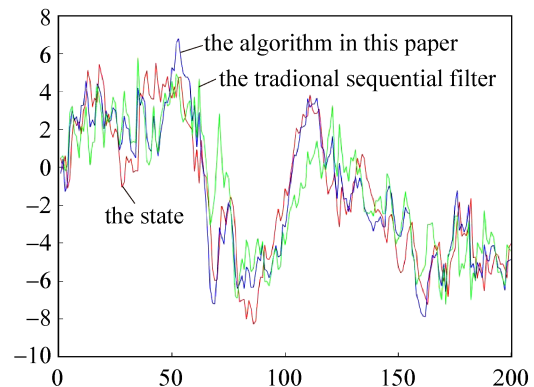


Fig. 4 Estimation curves of channel 2

From Fig. 6, we can see that the trace of error covariance of the filter with the buffer is much lower

than the trace of the error covariance of the filter without the buffer, which means that the method in this paper can reduce the impact of measurement delay. Meanwhile, the buffer length can also influence the performance of filter. As shown in Fig. 7, the performance of the filter with $L=5$ is better than the filter with $L=3$ i.e., the fusion center with longer buffer length will have better performance.

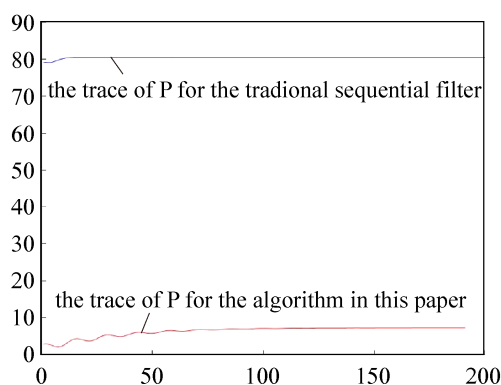


Fig. 5 Traces of error covariance of two filters

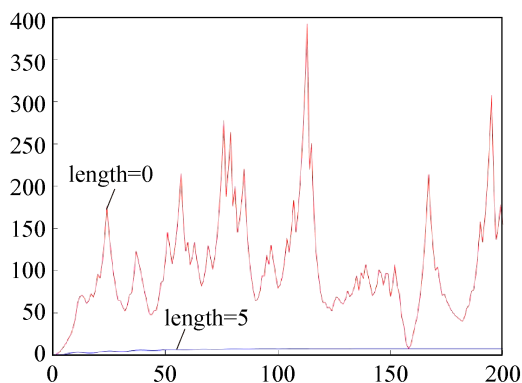


Fig. 6 Traces of the covariance of filter without buffer and $L=5$.

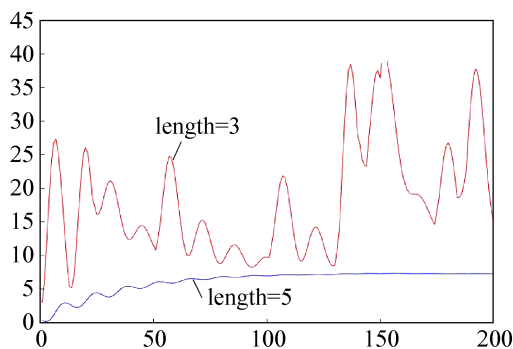


Fig. 7 Traces of the covariance of filter with $L=5$ and $L=3$.

From Fig. 8, we can know that the buffer with length $L=6$ cannot improve the performance too much, instead, it will increase the calculation burden of the system. Therefore, the buffer should choose the appropriate length. If the buffer length is too long, it will not significantly improve the performance, but increase the computational burden.

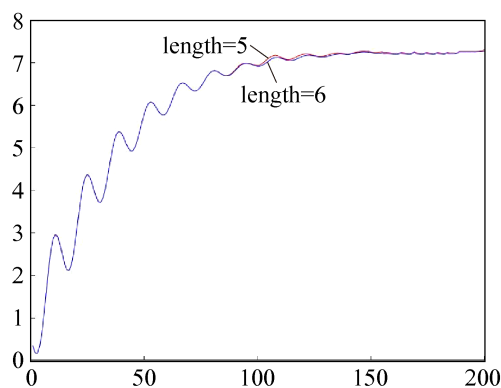


Fig. 8 Trace of the covariance of filter with $L=6$ and $L=5$.

4 Conclusions

For the wireless multi-sensor system, this paper considers two kinds of noise correlations and measurement delays. The correlations are removed by orthogonal transformation method first, then a sequential fusion algorithm with buffer is proposed to fuse the measurements from multi-channel and reduce the impact of measurements delay on the system at the same time.

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