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## Modeling and Simulation of Car-following on Expressway

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### Abstract

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### Keywords

car-following model, instrumented vehicle, stability analysis, FLOWSIM

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# Modeling and Simulation of Car-following on Expressway

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## 快速路车辆跟驰建模与仿真研究

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**摘要:** 针对快速路车辆较高速跟驰的现象, 将优化速度方程中固定的最小安全距离部分修改为可变的跟驰距离, 提出了新的车辆跟驰模型, 通过交通检测平台进行数据采集, 利用实测数据对可变最小跟驰距离和跟驰模型整体进行了标定与验证, 结果表明新模型能够更好的描述快速路上的较高速跟驰情况。利用经典控制理论中的 Hurwitz 稳定性判据对新模型进行了稳定性分析, 进行快速路跟驰仿真实验, 实验分为人造路网仿真和 FLOWSIM 快速路仿真两个部分。仿真结果表明新模型能够很好的应用于 FLOWSIM 中的快速路跟驰情况, 但是新模型的应用具有很高的限制要求。

**关键词:** 车辆跟驰理论; 移动试验车; 稳定性分析; FLOWSIM 仿真

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## Introduction

Car-following theory is used to describe how vehicles follow one another on a roadway. In



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car-following models, vehicles are regarded as discrete and interacting particles without overtaking to analyze traffic flow characteristics. So far, a variety of models have been developed, including safety distance model, stimulus-response model and fuzzy logic based model<sup>[1-8]</sup>. Among those models, the optimal velocity (OV) function is well known for its accuracy and rationality<sup>[3-4,9]</sup>. For decades the OV function has been extended by introducing

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generalized force, full velocity difference and lateral effects<sup>[10-14]</sup>. The improved OV model on two lanes has also been studied and the traffic wave has been simulated showing consistency with theoretical analysis<sup>[15]</sup>. This research proved that even without lane-changing behavior, the lateral impact could still have impact on the car-following procedure. However, the models mentioned above cannot be used to precisely reflect the car-following phenomenon in expressways with higher speed and larger headway (longer than 10 m), in which case the hyperbolic tangent part ( $\tanh$ ) in OV function remains as a constant. Therefore the safety headway distance parameter in OV function needs to be modified.

## 1 Improved OV model

### 1.1 Description of the improved model

The typical OV model is presented as

$$d^2x_n(t)/dt^2 = \alpha[V^{op}(\Delta x_n(t)) - v_n(t)] \quad (1)$$

where  $x_n(t)$  and  $v_n(t)$  are the position and velocity of the  $n$ th vehicle;  $\Delta x_n(t)$  is the headway distance between the  $n$ th and its leading vehicle;  $\alpha$  is the sensitivity parameter of the driver;  $V^{op}()$  is the optimal velocity function described as

$$V^{op}(\Delta x_n(t)) = v_{\max} / 2[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)] \quad (2)$$

where  $v_{\max}$  is the maximum velocity on a particular roadway;  $h_c$  means the safety headway distance.

However, as noticed in the study on expressway, the  $\Delta x_n(t) - h_c$  part is larger than on regular road which makes the  $\tanh()$  remain as 1. Thus, the following vehicle gets little or no influence despite  $\Delta x_n(t)$  is changing, which disobeys reality.

To avoid the aforementioned problem, the fixed parameter  $h_c$  in OV function is replaced by VSHD as the function of  $h_f = av_n(t) + b\Delta v_n(t) + h_c$ , where  $\Delta v_n(t)$  is the velocity difference between the  $n$ th and its leading vehicle. The physical demonstration of  $h_f$  is that the

acceptable safety headway distance of a driver is dynamic changing based on  $v_n(t)$  and  $\Delta v_n(t)$  level.

Referring to previous study<sup>[14-16]</sup>,  $\lambda\Delta v_n(t)$  is introduced to modified model, making the function as

$$\begin{cases} \frac{d^2x_n(t)}{dt^2} = \alpha[V_{\text{new}}^{op}(\Delta x_n(t)) - v_n(t)] + \lambda\Delta v_n(t) \\ V^{op}(\Delta x_n(t)) = \frac{v_{\max}}{2}[\tanh(\Delta x_n(t) - h_f) + \tanh(h_f)] \\ h_f = av_n(t) + b\Delta v_n(t) + h_c \end{cases} \quad (3)$$

### 1.2 Calibration and validation

The field data used for the study was collected by an instrumented vehicle equipped with radar detector system and inertial navigation system, by which  $\Delta x_n(t)$ ,  $\Delta v_n(t)$ ,  $v_n(t)$  and acceleration can be measured and recorded.

Data have been collected on expressways in Beijing in 2015. Drivers with different driving experiences ranging from 5 to 20 years were tested. Fig. 1 shows the graph of the experiment.

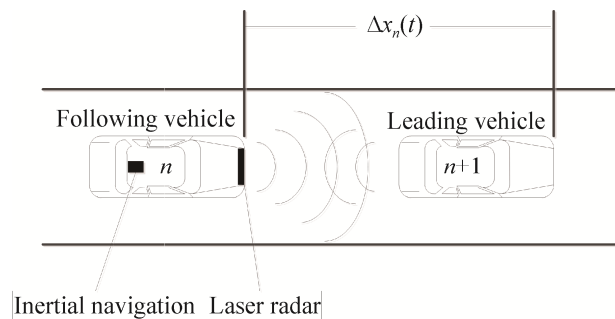


Fig. 1 Data collection experiment by instrumented vehicle

For the calibration of VSHD, the driver was requested to drive at different speeds from 30 km/h to 90 km/h. Data was recorded when the driver felt safety distance was reached. As expected, VSHD is related with  $v_n(t)$  and  $\Delta v_n(t)$ . The calibrated VSHD formula is presented as

$$h_f = 1.42v_n(t) + 4.21\Delta v_n(t) + 3.41$$

As  $v_{\max}$  is 90 km/h, we have the OV function as

$$V^{op}(\Delta x_n(t)) = 45[\tanh(\Delta x_n(t) - 1.42v_n(t) - 4.21\Delta v_n(t) - 3.41) + \tanh(1.42v_n(t) + 4.21\Delta v_n(t) + 3.41)]$$

Then the parameters of acceleration equation can be calibrated with the collected data as

$$d^2x_n(t)/dt^2 = 0.0037[V^{op}(\Delta x_n(t)) - v_n(t)] + 0.11\Delta v_n(t)$$

The new model is validated with test data. Fig. 2 shows the comparison results of acceleration change process, indicating the new model can better match the field situation. We can also find that real data curves always rise or fall more sharply. That is caused by the assumption that when a driver is actually driving he would press the brake or the gas pedal without transition period, while the model is a smooth process.

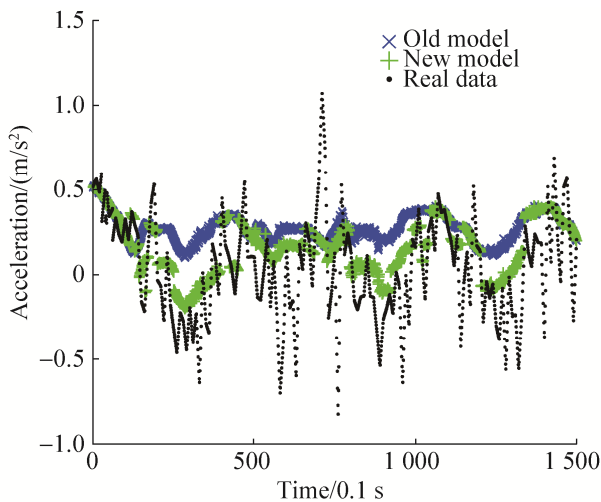


Fig. 2 Comparison between new model and typical model

## 2 Stability analysis

According to stability analysis method [16-18], the stable condition of the modified car-following model can be shown as

$$\begin{cases} \frac{d^2x_n(t)}{dt^2} = \alpha[V^{op}(\Delta x_n(t)) - v_n(t)] + \lambda\Delta v_n(t) \\ \frac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t) \end{cases} \quad (4)$$

where  $y_n(t) = x_{n+1}(t) - x_n(t)$ . We assume the first vehicle is not influenced by others and runs constantly at

speed  $v$ , and then the steady state of the system is

$$[v_n^*(t), y_n^*(t)]^T = [v, V^{op-1}(v)]^T \quad (5)$$

The linearized system of (4) can be calculated around steady state (5) as

$$\begin{cases} \frac{d\delta v_n(t)}{dt} = \alpha[\delta y_n(t)\Lambda - \delta v_n(t)] + \lambda(\delta v_{n+1}(t) - \delta v_n(t)) \\ \frac{d\delta y_n(t)}{dt} = \delta v_{n+1}(t) - \delta v_n(t) \end{cases} \quad (6)$$

where  $\delta v_n(t) = v_n(t) - v$ ,  $\delta y_n(t) = y_n(t) - V^{op-1}(v)$ , and the slope  $\Lambda = dV^{op}(\Delta y_n(t))/dy_n(t)|_{y_n(t)=V^{op-1}(v)}$ .

After Laplace transformation, (6) is shown as

$$\begin{bmatrix} V_n(s) \\ Y_n(s) \end{bmatrix} = \frac{V_{n+1}(s)}{p(s)} \begin{bmatrix} s & \alpha\Lambda \\ -1 & s + \alpha + \lambda \end{bmatrix} \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \quad (7)$$

where  $V_n(s) = L(\delta v_n(t))$ ,  $Y_n(s) = L(\delta y_n(t))$ ,  $L()$  is the Laplace transformation and the characteristic polynomial is  $p(s) = s^2 + s\lambda + s\alpha + \alpha\Lambda$ . Then the transfer function is

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{p(s)} \begin{bmatrix} s & \alpha\Lambda \\ -1 & s + \alpha + \lambda \end{bmatrix} \begin{bmatrix} \lambda \\ 1 \end{bmatrix} = \frac{s\lambda + \alpha\Lambda}{p(s)} = \frac{s\lambda + \alpha\Lambda}{s^2 + s\lambda + s\alpha + \alpha\Lambda} \quad (8)$$

The traffic jam will never happen in this system if  $p(s)$  is stable and  $\|G(s)\|_\infty \leq 1$ .

As  $d^2x_n(t)/dt^2$  is in positive correlation with  $[V^{op}(\Delta x_n(t) - v_n(t))]$ , we have  $\alpha > 0$ . Because of the OV function characteristic monotonously increasing (i.e.  $\Lambda > 0$ ), the condition for  $p(s)$  to be stable is  $\lambda > 0$ .

Then we consider  $\|G(s)\|_\infty \leq 1$  which is

$$\begin{aligned} \|G(s)\|_\infty &= \sup |G(j\omega)| \leq 1, \\ |G(j\omega)|^2 &= |G(-j\omega)G(j\omega)| = \\ &= (\alpha^2\Lambda^2 + \omega^2\lambda^2) / [(\alpha\Lambda - \omega^2)^2 + (\alpha + \lambda)^2\omega^2] \leq 1 \end{aligned}$$

The sufficient condition can be obtained as

$$\alpha^2 + 2\alpha\lambda + \omega^2 - 2\alpha\Lambda \geq 0, \omega \in [0, \infty) \quad (9)$$

which can be written as

$$0 \leq \Lambda \leq \alpha^2 + 2\alpha\lambda/2\alpha, (\lambda > 0) \quad (10)$$

### 3 Numerical simulation tests

We consider a 100-vehicle system running on a 6km expressway without overtaking. In the simulations, the initial values are set  $v_n(0)=60$  km/h and  $v_{\max}=90$  km/h. It is assumed all vehicles have the same parameters. In the simulation process the leading vehicle has a random fluctuation in acceleration/deceleration. The following Fig. 3 shows the trajectory of vehicles.

Fig. 3 shows the comparison between new model and typical OV model in high speed car-following. Fig. 3(a) illustrates that with a larger distance gap but higher speed, vehicles in new model

are more sensitive to the variation of leading vehicle's acceleration/deceleration and several traffic waves are formed during the process, while in Fig. 3(b) the vehicles in typical model just get slightly influenced.

We consider a situation where the leading vehicle stops suddenly for a short time and Fig. 4 describes the comparison between steady state and non-steady state of new model. Fig. 4(a) means when stability condition is met for new model, the vehicles can adjust themselves gradually although the headway fluctuation appears in the beginning. However, in Fig. 4(b) the heavy congestion continues through the simulation and the trajectories of some vehicles overlap.

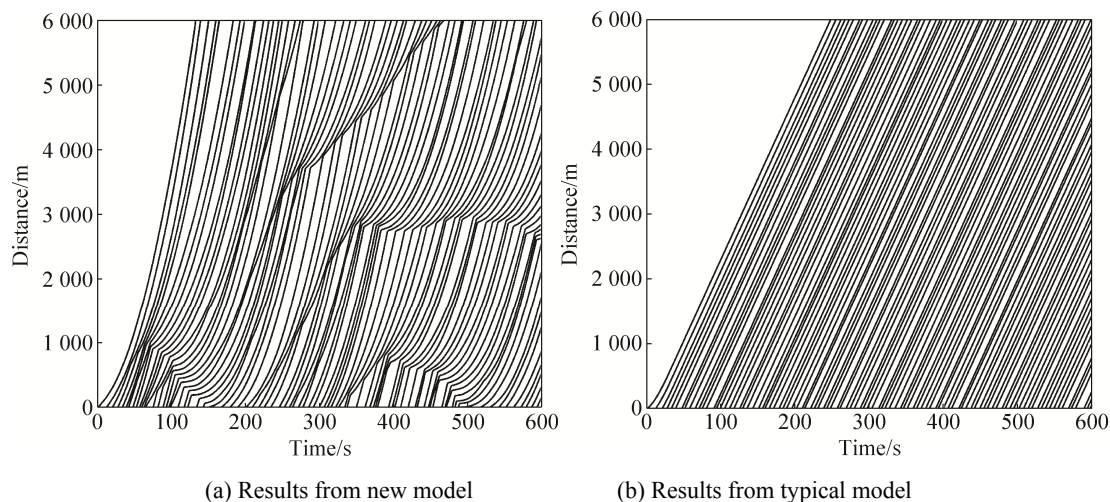


Fig. 3 Comparison between new model and typical model

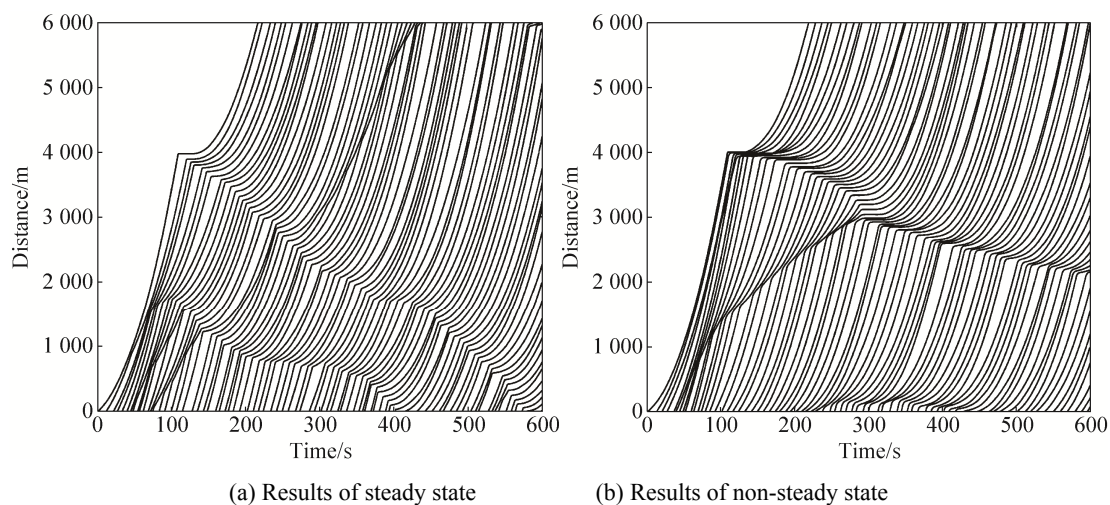


Fig. 4 Comparison between steady state and non-steady state

## 4 Application in the system of FLOWSIM

### 4.1 Simulation system of FLOWSIM

In last section, the new car-following model is only simulated in a simple synthetic scenario, where there are many limitations. As a result, one could argue that the test is far from real world and cannot be persuasive. Therefore, it is necessary to implement the new model into a more realistic scenario. However, the theory cannot be verified in real world as the drivers' stochastic behavior cannot be formulated. Thus, in this paper, the microscopic traffic simulation system, FLOWSIM, is introduced to act as the real world, and the improved OV theory is integrated into the system as the car-following module.

There are many researches focusing on the application of various traffic simulation systems to replicate the field traffic system and implement related studies. A study introduced AIMSUN to investigate if adjusting a selected set of simulation parameters can improve the emissions estimated from simulated trajectories on a signalized arterial corridor<sup>[19]</sup>. The mesoscopic traffic simulator, DynaMIT-P, has also been developed successfully to represent the complex traffic characteristics in Beijing with the Path-size Logit route choice model and other improvements<sup>[20]</sup>. A previous research utilized VISSIM to investigate the performance of a stochastic model established to dynamically optimize the green times at isolated intersections under various scenarios<sup>[21]</sup>. As for the simulation system in this paper, FLOWSIM is a fuzzy logic based motorway traffic simulation model, firstly developed at University of Southampton, UK. It has been successfully applied in many cities in Europe and

well calibrated and validated. From this century on, the FLOWSIM team has moved to Beijing, China and began to calibrate the system parameters based on data collected from Beijing<sup>[8]</sup>. The FLOWSIM has been applied for mixed traffic flow management in mid-size cities and for isolated signalized intersection assessment in large cities. The applications show promising results, providing a new management method for traffic authorities and research scientists<sup>[22]</sup>. In Shenzhen, the FLOWSIM is utilized to study the Transportation Demand Management (TDM) solutions including the congestion charge policy to mitigate traffic jams in large city<sup>[23]</sup>. The simulation tests provide useful statistical assessment results and show the advantages of introducing the FLOWSIM to analyze the policy before implementation. Furthermore, the FLOWSIM can describe the mixed traffic phenomenon in China by integrating the bicycle simulation module. It has been successfully used to simulate the bicycle multi-phase crossing problem at intersections in Beijing, China<sup>[24]</sup>. The simulation results show that bicycle multi-phase crossings with proper signal operations can reduce bicycle delays dramatically with only a minor effect on vehicle delay, and can reduce the number of vehicle stops. The FLOWSIM has also been used in many other cities in China to solve traffic problems<sup>[25-27]</sup>. All the application results conclude that the FLOWSIM can represent the real traffic system for further researches.

As aforementioned above, the key algorithm in FLOWSIM is fuzzy logic based models. In this logic, the driver's behaviors are divided into a set of new fuzzy parameters by developing a series of membership functions. The objective of the FLOWSIM is to substitute typical statistical models by fuzzy membership functions to better reflect

driver's behavior. Commonly a microscopic traffic simulation system contains the car-following module and the lane-changing module. However, in this study, only the car-following module is the main focus. In FLOWSIM, the car-following module has two important parameters deciding the driver's behavior, including (a) the relative speed (DV) between the leading vehicle and the following one, and (b) the

ratio of distance between the two vehicles to the driver's desired gap (DSSD)<sup>[22,28]</sup>. The fuzzy logic for these two parameters is shown in Tab. 1, where the response represents the following vehicle's desired behavior. To be noted, the DV and DSSD have an additive effect. For example, if the following driver feels he is on stage V2 and S4, then the response is no action<sup>[28]</sup>.

Tab. 1 Fuzzy logic for car-following module in FLOWSIM

DV (relative speed)	DSSD (distance divergence)	Response (acceleration rate)
Opening fast (V1)	Much to far (S1)	Strong acceleration
Opening (V2)	Too far (S2)	Light acceleration
About zero (V3)	Satisfied (S3)	No action
Closing (V4)	Too close (S4)	Light deceleration
Closing fast (V5)	Much too close (S5)	Strong deceleration

In this study, the above fuzzy logic based car-following module is replaced by the improved OV model developed in the above sections, while other modules in FLOWSIM remain unchanged. To validate the feasibility of new car-following module, the modified system is tested under the expressway scenario.

## 4.2 Simulation experiments of modified FLOWSIM

In this paper, the survey site is a 1.7 km long arterial between Deshengmenqiao and Madianqiao in the downtown Beijing. The section is chosen as the simulation study area for the advantages of no signalized intersections and thus could represent the general condition in urban area of Beijing.

High-resolution traffic historical data are essential in this research. On the study section, traffic flow rate, mean speed and occupancy are recorded in 2 min aggregation interval using detectors and archived by the Beijing Traffic Management Bureau (BTMB). The research period in this paper is from

2014-06-08. Fig. 5 shows the study area. It is noticed that the initial interval of traffic historical data is in a very high resolution, leading to a high computation complexity and low simulation effectiveness. As a result, the aggregation of 10 min is introduced in this simulation study and previous reference shows that this resolution can still accurately capture the traffic flow characteristics<sup>[29]</sup>. So a 10 min time interval is assumed in this research to process traffic detector data, using weighted average method by flow rate, given as (11) and (12).

$$\text{Flow}_{10\text{min}} = \sum \text{Flow}_{2\text{min}} \quad (11)$$

$$\text{Speed}_{10\text{min}} = \frac{\sum (\text{Flow}_{2\text{min}} \times \text{Speed}_{2\text{min}})}{\text{Flow}_{10\text{min}}} \quad (12)$$

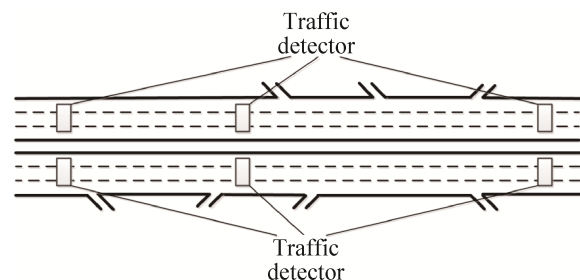


Fig. 5 Structure of the study expressway.



The 10 min interval is also set in the FLOWSIM as the data collection resolution. The field data obtained from traffic detectors are set as the reference. The comparison between simulated data using original FLOWSIM and the field data is regarded as the benchmark. Then we also analyse the comparison between simulated data using OV model integrated FLOWSIM between the field data to find out whether the performance is improved. All the simulation experiments are divided into two groups: (a) morning peak hours in weekday; (b) regular hours in weekday. To evaluate the effectiveness of the proposed model, we use 2 performance measurements: the Mean Absolute Percentage Error (MAPE) and the Root Mean Square Error (RMSE), expressed in (13) and (14).

$$\text{MAPE} = \frac{1}{N} \sum_{i=1}^N \frac{|x_i - y_i|}{x_i} \quad (13)$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - y_i)^2} \quad (14)$$

where  $x_i$  is the archived field data,  $y_i$  is the simulated data, and  $N$  is the sample size.

The comparison results of the simulated data against the field data from fuzzy logic based and improved OV based FLOWSIM are listed in Tab. 2, Tab. 3 for regular hours and peak hours, respectively.

From Tab. 2, it is observed that the improved OV model based FLOWSIM could perform better than the fuzzy logic based one during the regular hours. All the performance measurements by improved FLOWSIM for both the speed and the flow count show advantages against the original version of FLOWSIM. Also, it is noted that the improvement of flow count is larger than that of speed. However, from Tab. 3, the outcome shows the opposite conclusion. For peak hours on study expressway, the improved OV model FLOWSIM is outperformed by

the original fuzzy logic based version. The same as in Tab. 2, the discrepancies of flow count are larger than those of speed. The different outcomes from these two tables can be explained by the fact that during peak hours the traffic flow on expressway is under congestion, which will cause the vehicles travel at a low speed making the improved OV model inappropriate to utilize. One can tell from these results that the new model based FLOWSIM can only outperform the original version when the vehicles are at a high speed. Thus even on expressways, the new model cannot be used for all the time. An intelligent switch strategy should be included in FLOWSIM to automatically change between these two car-following modules according to the current travel speed on expressways.

Given the simulation experiments, it is concluded that the improved OV model could be used in FLOWSIM to increase the simulation accuracy.

Tab. 2 Comparison between the two modules for regular hours

Measurement	Fuzzy logic based	Improved OV based	Increment/%
MAPE <sub>speed</sub>	0.18	0.17	5.56
MAPE <sub>flow</sub>	0.25	0.23	8
RMSE <sub>speed</sub>	9.67	9.54	1.34
RMSE <sub>flow</sub>	17.84	16.01	10.26

Tab. 3 Comparison between the two modules for peak hours

Measurement	Fuzzy logic based	Improved OV based	Decrement/%
MAPE <sub>speed</sub>	0.20	0.25	25
MAPE <sub>flow</sub>	0.31	0.42	35.48
RMSE <sub>speed</sub>	10.03	14.84	47.96
RMSE <sub>flow</sub>	22.91	34.76	51.72

## 5 Conclusion

In this paper we proposed a new car-following model considering the variable safety headway distance based on the typical OV model. The new

model is calibrated, validated with data collected by instrumented vehicle and the stability condition is analyzed by applying the control theory. Furthermore, the numerical simulations are given, and results show consistency with the theoretical study. Finally, the application of the improved OV model is investigated in the microscopic traffic simulation system of FLOWSIM. A field expressway in Beijing is selected to implement simulation experiments. The comparison results between improved OV model based FLOWSIM and its original version show that the modified FLOWSIM could outperform the other one only when the traffic flow is moving at a high speed. In conclusion, the modified model could reflect the phenomenon that following vehicles can easily get affected on expressways with high speed although the headway distance is large. However, an intelligent switch strategy should be included in FLOWSIM to automatically change between these two car-following modules according to the current travel speed on expressways.

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