[Journal of System Simulation](https://dc-china-simulation.researchcommons.org/journal)

Volume 30 | Issue 11 Article 4

1-4-2019

Modeling and Simulation of Travelers' Route Choice Behavior Based on Disequilibrium Theory

Zhongxiang Huang
1. School of Traffic and Transportation Engineering, Changsha University of Science and Technology, Changsha 410114, China; ;

Jianhui Wu
1. School of Traffic and Transportation Engineering, Changsha University of Science and Technology, Changsha 410114, China; ;2. School of Information Science and Technology, Hunan Institute of Science and Technology, Yueyang 414006, China;

Aiwu Kuang
1. School of Traffic and Transportation Engineering, Changsha University of Science and Technology, Changsha 410114, China; ;

Zhang Sheng
1. School of Traffic and Transportation Engineering, Changsha University of Science and Technology, Changsha 410114, China; ;

See next page for additional authors

第 30 卷第 11 期
2018 年 11 月

系统仿真学报©
Journal of System Simulation

Vol. 30 No. 11
Nov., 2018

Modeling and Simulation of Travelers' Route Choice Behavior Based on Disequilibrium Theory

Huang Zhongxiang[1], *Wu Jianhui*[1,2*], *Kuang Aiwu*[1], *Zhang Sheng*[1], *Xu Yan*[2]

(1. School of Traffic and Transportation Engineering, Changsha University of Science and Technology, Changsha 410114, China;
2. School of Information Science and Technology, Hunan Institute of Science and Technology, Yueyang 414006, China)

Abstract: Considering the influence of heterogeneous travel on traffic flow distribution, *the travelers' route choice behaviors are classified into three types: price regulation, quantity regulation and price-quantity regulation. Three day-to-day dynamical stochastic assignment models based on the disequilibrium theory are established, and the existence, uniqueness, and stability of the solutions to these models are proved.* Through model simulation, it proved that each path flow in these three models can converge to a steady state after a finite travel adjustment, and the day-to-day dynamical stochastic assignment model under price-quantity regulation can simulate the influence of heterogeneous travel behaviors on the distribution characteristic and dynamic evolution of traffic flow.

Keywords: route choice behavior; stochastic traffic assignment; disequilibrium theory; price regulation; quantity regulation

基于非均衡理论的出行者路径选择行为建模与仿真

黄中祥[1]，伍建辉[1,2*]，况爱武[1]，张生[1]，许艳[2]

(1. 长沙理工大学 交通运输工程学院，湖南 长沙 410114；2. 湖南理工学院 信息科学与工程学院，湖南 岳阳 414006)

摘要：考虑出行异质性对交通流分布的影响，*基于非均衡理论将出行者路径选择行为分为价格调节、数量调节和价格—数量调节三类，建立三种逐日动态随机分配模型，证明了逐日动态随机分配模型解的存在性、唯一性和稳定性*。通过模型仿真，验证了三种逐日动态随机分配模型经有限次出行调整后，各路径流量均能收敛到稳定状态，价格—数量调节逐日动态随机分配模型能够模拟出行异质性对交通流分布形态与动态演化的影响。

关键词：路径选择行为；随机交通分配；非均衡理论；价格调节；数量调节

中图分类号：U491.2 文献标识码：A 文章编号：1004-731X (2018) 11-4067-12

DOI: 10.16182/j.issn1004731x.joss.201811004

Introduction

Traffic assignment is an important step in urban traffic planning. The key to traffic assignment modeling is making scientific, reasonable, and realistic assumptions on the route choice behavior of travelers. Wardrop's user equilibrium principle[1] is the basis of traffic assignment modeling. It contains three basic behavior assumptions: one is complete information, that is, travelers can obtain all traffic network information; second is complete rationality,

Received: 2018-05-11 Revised: 2018-06-19;
Fundation: National Natural Science Foundation of China (51338002, 51408058), Science and Technology Program of Hunan Province (2016TP1021);
Biography: Huang Zhongxiang (1965-), Male, Miluo Hunan, Doctor, Professor, His main research interest is transportation planning and management.

that is, travelers can choose the travel route with minimum travel time; and third is homogeneity, that is, different travelers have exactly the same travel characteristics and preferences; in other words, the decisions are made on the same criterion. In the literature, some traffic assignment models have partially relaxed the above assumptions. For example, the stochastic user equilibrium model[2-4] relaxes the complete information assumption, whereas the traffic assignment model based on the prospect theory[5-7] and the traffic assignment model based on the regret theory[8-10] relax the complete rationality assumption.

In recent years, the heterogeneity of travelers' route choice behavior has also attracted the attention of scholars. Peeta and Mahmassani[11] assumed that the advanced traveler information systems (ATIS) or advanced traffic management systems (ATMS) have current time intervals and short-term and medium-term traffic information between OD pairs. They proposed a system framework to solve a multi-user, real-time traffic assignment. Florian et al.[12] classified travelers into different user types according to the transit vehicles they use, and studied the network equilibrium problem under a multi-class, multi-mode variable demand. Through experiments and studies, Brown et al.[13] found that travelers would also make different travel decisions if their expected levels change even in the same traffic scenarios. Ramos et al.[14] discussed the role of heterogeneity in travelers' behavior and proposed two distinct model frameworks: one accounting for heterogeneity and another considering no heterogeneity. Zockaie et al.[15] studied the influence of congestion charging on different travelers' route choice behaviors and established a multi-criteria dynamic user equilibrium traffic assignment model.

The disequilibrium theory indicated that traders could not only get the price signal but also obtain the quantity signal in the market. Huang et al.[16] believed that travelers would consider not only the travel time (price) of the current route but also the vehicle flow (quantity) obtained from traffic information and historical experience when making decisions on travel routes. They proposed a new idea of disequilibrium transportation planning based on the disequilibrium theory. In actual traffic management, congestion charge[17-19] and parking charge[20-22] can change the travel decisions of some travelers by a price factor. Furthermore, ramp control[23-25] and public transport departure frequency optimization[26-29] can change the travel decision of some travelers by a quantity factor. Therefore, travelers are subject to the two constraints of price and quantity. Based on this idea, this study regards travel time as price and regards free driving opportunity and driving comfort as quantity. Meanwhile, we classify the travelers' route choice behavior into three types: full compliance with travel time on optional routes, full compliance with free driving opportunity and driving comfort on optional routes, and comprehensive consideration of travel time, free driving opportunity, and driving comfort on optional routes. In addition, we define these three types of travelers' route choice behavior as fully compliant with price regulation, quantity regulation, and price-quantity regulation respectively. We use the path residual capacity to describe the free driving opportunity and driving comfort on the optional routes and analyze the influence of heterogeneous travel behaviors on the distribution characteristic and dynamic evolution of traffic flow.

The remainder of this paper is organized as follows. In the next section, we classify the travelers' route choice behavior into three types and establish

https://dc-china-simulation.researchcommons.org/journal/vol30/iss11/4
DOI: 10.16182/j.issn1004731x.joss.201811004

three day-to-day dynamical stochastic assignment models based on the disequilibrium theory. Section 2 presents the proof of existence, uniqueness, and stability of the solution to the day-to-day dynamical stochastic assignment model. The properties of three models are demonstrated in Section 3 and the conclusions of this study are outlined in Section 4.

1 Modeling

Different route choice criteria naturally lead to different traffic flow distributions. Based on the disequilibrium theory, we classify the travelers' route choice behavior into three types, and establish three day-to-day dynamical stochastic assignment models under price regulation, quantity regulation, and price-quantity regulation, respectively.

1.1 Travel route choice behavior under price regulation

Suppose that travelers choose a travel route according to the travel time of the optional routes; we call this type of travelers' route choice behavior as fully compliant with price regulation. The relationship between link flow f_a and path flow h_r can be represented as

$$f_a(n) = \sum_{w\in W}\sum_{r\in R_w}\delta_{ar}h_r(n),\quad \forall a\in A \tag{1}$$

where W denotes the set of OD pairs, A is the set of links, R_w is the set of paths connecting OD pair w, and δ_{ar} is the link-path incidence; specifically, $\delta_{ar}=1$ if $a\in r$ and $\delta_{ar}=0$ otherwise. The relationship between path-travel time c_r, and link-travel time c_a can be represented as

$$c_r(h_r(n)) = \sum_{a\in A}\delta_{ar}c_a(f_a(n)) \tag{2}$$

We formulate the expected travel time at day n as the weighted sum of the expected travel time and path-travel time at the previous day n–1. Then the expected travel time C_r on path r at day n can be written as

$$C_r(n) = \kappa C_r(n-1) + (1-\kappa)c_r(h_r(n-1)) \tag{3}$$

where $\kappa\ (0\leqslant\kappa<1)$ denotes a constant weight corresponding to travelers, which reflects the preference between path-travel time and expected travel time by travelers.

Suppose that d_w is the travel demand between OD pair w and ε_r is a random term with $E(\varepsilon_r)=0$ denoting the traveler's perception error. According to the stochastic user equilibrium assignment model, the choice probability p_r on path r at day n can be represented as

$$p_r(n) = P(C_r(n)+\varepsilon_r \leqslant \bigcup_{k\in R_w}(C_k(n)+\varepsilon_k)) = \frac{1}{1+\sum_{k\neq r}e^{-\phi(C_k(n)-C_r(n))}},\quad \forall k,r\in R_w \tag{4}$$

where $\sum_{r\in R_w}p_r(n)=1$, $\phi\ (\phi>0)$ reflects the sensitivity of travelers to the expected travel time. The higher the value of ϕ, the more sensitive the travelers, and vice versa. The path flow h_r at day n can be written as

$$h_r(n) = d_w p_r(n) \tag{5}$$

Hence, the day-to-day dynamical stochastic assignment model under price regulation can be written as

$$\begin{cases} p_r(n) = \dfrac{1}{1+\sum_{k\neq r}e^{-\phi(C_k(n)-C_r(n))}} \\ h_r(n) = d_w p_r(n) \\ C_r(n) = \kappa C_r(n-1) + (1-\kappa)c_r(h_r(n-1)) \end{cases} \tag{6}$$

1.2 Travel route choice behavior under quantity regulation

Suppose that travelers cannot obtain travel time information or do not choose a travel route according to travel time but choose it according to vehicle flow (quantity) on the optional routes. We call this type of traveler's route choice behavior as fully compliant with quantity regulation.

Published by Journal of System Simulation, 2018

In this study, we use the link-residual capacity to describe the vehicle flow on the link. The more the residual capacity, the more the free driving opportunity and the better the driving comfort. Suppose that K_a is the capacity on link a, the residual capacity v_a on link a can be represented as

$$v_a(f_a(n)) = K_a - f_a(n) \tag{7}$$

The relationship between path-residual capacity v_r and link-residual capacity v_a can be represented as

$$v_r(h_r(n)) = \min_{a\in r}\{v_a(f_a(n))\} \tag{8}$$

We formulate the expected residual capacity at day n as the weighted sum of the expected residual capacity and path-residual capacity at the previous day n–1. Then the expected residual capacity V_r on path r at day n can be written as

$$V_r(n) = \eta V_r(n-1) + (1-\eta)v_r(h_r(n-1)) \tag{9}$$

where η $(0 \leqslant \eta < 1)$ denotes a constant weight corresponding to travelers, which reflects the preference between path-residual capacity and expected residual capacity by travelers. According to the stochastic user equilibrium assignment model, the choice probability p_r on path r at day n can be represented as

$$p_r(n) = P(V_r(n) + \varepsilon_r \geqslant \bigcup_{k\in R_w}(V_k(n) + \varepsilon_k)) = \frac{1}{1+\sum_{k\neq r} e^{\varphi(V_k(n)-V_r(n))}},\quad \forall k, r \in R_w \tag{10}$$

where φ $(\varphi > 0)$ reflects the sensitivity of travelers to the expected residual capacity. Then, the day-to-day dynamical stochastic assignment model under quantity regulation can be written as

$$\begin{cases} p_r(n) = \dfrac{1}{1+\sum_{k\neq r} e^{\varphi(V_k(n)-V_r(n))}} \\ h_r(n) = d_w p_r(n) \\ V_r(n) = \eta V_r(n-1) + (1-\eta)v_r(h_r(n-1)) \end{cases} \tag{11}$$

1.3 Travel route choice behavior under price-quantity regulation

Suppose that travelers choose a travel route according to the comprehensive travel cost weighted by travel time and residual capacity on the optional routes. We call this type of traveler's route choice behavior as fully compliant with price-quantity regulation.

We formulate the comprehensive travel cost s_r at day n as the weighted sum of path-travel time c_r and path-residual capacity v_r at day n. Then the comprehensive travel cost s_r on path r at day n can be written as

$$s_r(h_r(n)) = \lambda c_r(h_r(n)) - (1-\lambda)v_r(h_r(n)) \tag{12}$$

where λ $(0 \leqslant \lambda \leqslant 1)$ denotes the weighting factor, which reflects the preference between path-travel time and path-residual capacity by travelers.

We formulate the expected comprehensive travel cost S_r at day n as the weighted sum of the expected travel time C_r and expected residual capacity V_r at day n. Then the expected comprehensive travel cost S_r on path r at day n can be written as

$$S_r(n) = \lambda C_r(n) - (1-\lambda)V_r(n) \tag{13}$$

According to the stochastic user equilibrium assignment model, the choice probability p_r on path r at day n can be represented as

$$p_r(n) = P(S_r(n) + \varepsilon_r \leqslant \bigcup_{k\in R_w}(S_k(n) + \varepsilon_k)) = \frac{1}{1+\sum_{k\neq r} e^{-\theta\{[\lambda C_k(n)-(1-\lambda)V_k(n)]-[\lambda C_r(n)-(1-\lambda)V_r(n)]\}}},\quad \forall k, r \in R_w \tag{14}$$

where θ $(\theta > 0)$ reflects the sensitivity of travelers to the expected comprehensive travel cost. Using the above Eqs. (3), (5), (9) and (14), the day-to-day dynamical stochastic assignment model under price-quantity regulation can be written as

$$\begin{cases} p_r(n) = \\ \dfrac{1}{1+\sum\limits_{k\neq r} e^{-\theta\{\lambda C_k(n)-(1-\lambda)V_k(n)-[\lambda C_r(n)-(1-\lambda)V_r(n)]\}}} \\ h_r(n) = d_w p_r(n) \\ C_r(n) = \kappa C_r(n-1) + (1-\kappa)c_r(h_r(n-1)) \\ V_r(n) = \eta V_r(n-1) + (1-\eta)v_r(h_r(n-1)) \end{cases} \quad (15)$$

2 Properties analysis of model

It should be noted that model (15) becomes a day-to-day dynamical stochastic assignment model under price regulation if $\lambda = 1$, and it becomes a day-to-day dynamical stochastic assignment model under quantity regulation if $\lambda = 0$; hence, we only analyze the steady state of model (15).

2.1 Existence and uniqueness of model solution

Theorem 1. If link-travel time is a continuous and strictly monotonically increasing function of link flow, and link-residual capacity is a continuous and strictly monotonically decreasing function of link flow, then model (15) has a unique solution under a fixed travel demand.

Proof. We first prove the existence of the model solution. Because the travel demand is bounded, the set of path flows is a nonempty bounded closed convex set. According to the definition of path-travel time and path-residual capacity, the set of path-travel time and the set of path-residual capacity are also nonempty bounded closed convex sets. Hence, model (15) continuously maps nonempty bounded closed convex sets to itself. According to Brouwer's fixed point theory[30], model (15) has at least one solution.

Second, we prove the uniqueness of the model solution. Suppose that the expected travel time $C_r(n) = C_r(n-1) = C_r^*$, expected residual capacity $V_r(n) = V_r(n-1) = V_r^*$ and path flow $h_r(n) = h_r(n-1) = h_r^*$ at the steady state of model (15), we have

$$C_r^* = c_r(h_r^*) \quad (16)$$

$$V_r^* = v_r(h_r^*) \quad (17)$$

$$h_r^* = \frac{d_w}{1+\sum\limits_{k\neq r} e^{-\theta\{\lambda C_k^*-(1-\lambda)V_k^*-[\lambda C_r^*-(1-\lambda)V_r^*]\}}} \quad (18)$$

The above fixed point problems can be transformed into the following variational inequality:

$$\sum_{w\in W}\sum_{r\in R_w}[\lambda c_r(h_r^*) - (1-\lambda)v_r(h_r^*) + \frac{1}{\theta}\ln h_r^*](h_r - h_r^*) \geqslant 0 \quad (19)$$

Suppose that model (15) has two different solutions $(C_r^{1*}, V_r^{1*}, h_r^{1*})$ and $(C_r^{2*}, V_r^{2*}, h_r^{2*})$. According to Eq. (19), we have

$$\sum_{w\in W}\sum_{r\in R_w}[\lambda c_r(h_r^{1*}) - (1-\lambda)v_r(h_r^{1*}) + \frac{1}{\theta}\ln h_r^{1*}](h_r^{2*} - h_r^{1*}) \geqslant 0 \quad (20)$$

$$\sum_{w\in W}\sum_{r\in R_w}[\lambda c_r(h_r^{2*}) - (1-\lambda)v_r(h_r^{2*}) + \frac{1}{\theta}\ln h_r^{2*}](h_r^{1*} - h_r^{2*}) \geqslant 0 \quad (21)$$

By combining Eq. (21) with (20), we obtain

$$\left\{\sum_{w\in W}\sum_{r\in R_w}\lambda(c_r(h_r^{1*}) - c_r(h_r^{2*})) - (1-\lambda)(v_r(h_r^{1*}) - v_r(h_r^{2*})) + \sum_{w\in W}\sum_{r\in R_w}\frac{1}{\theta}(\ln h_r^{1*} - \ln h_r^{2*})\right\}(h_r^{1*} - h_r^{2*}) \leqslant 0 \quad (22)$$

Because link-travel time is strictly monotonically increasing with link flow, we know that $c_r(h_r)$ is strictly monotonically increasing with h_r. Similarly, we also know that $v_r(h_r)$ is strictly monotonically decreasing with h_r. Additionally, $\ln h_r$ is strictly monotonically increasing with h_r; thus, we have

$$\left\{\sum_{w\in W}\sum_{r\in R_w}\lambda(c_r(h_r^{1*}) - c_r(h_r^{2*})) - (1-\lambda)(v_r(h_r^{1*}) - v_r(h_r^{2*})) + \sum_{w\in W}\sum_{r\in R_w}\frac{1}{\theta}(\ln h_r^{1*} - \ln h_r^{2*})\right\}(h_r^{1*} - h_r^{2*}) > 0 \quad (23)$$

Eq. (22) is inconsistent with Eq. (23); therefore, model (15) has a unique solution.

2.2 Stability of model solution

Suppose that there exist m $(r=1,\cdots,m)$ paths between OD pair w, and $C_r(n)=C_r^*$, $V_r(n)=V_r^*$, $h_r(n)=h_r^*$ at the steady state of model (15). Additionally, we make the following definitions:

$$c_r' = \left.\frac{\partial c_r(n)}{\partial h_r(n)}\right|_{(C_r^*,V_r^*,h_r^*)} \tag{24}$$

$$\begin{cases} p_{rC_r}' = \dfrac{\partial p_r(n)}{\partial C_r(n)} = \\ -\theta\lambda \left.\dfrac{\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}}{\left(1+\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \\ p_{rC_k}' = \dfrac{\partial p_r(n)}{\partial C_k(n)} = \\ \theta\lambda \left.\dfrac{e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}}{\left(1+\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \end{cases} \tag{25}$$

$$\begin{cases} p_{rV_r}' = \dfrac{\partial p_r(n)}{\partial V_r(n)} = \\ \theta(1-\lambda) \left.\dfrac{\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}}{\left(1+\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \\ p_{rV_k}' = \dfrac{\partial p_r(n)}{\partial V_k(n)} = \\ -\theta(1-\lambda) \left.\dfrac{e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}}{\left(1+\sum\limits_{k\neq r} e^{-\theta\{[\lambda C_k-(1-\lambda)V_k]-[\lambda C_r-(1-\lambda)V_r]\}}\right)^2}\right|_{(C_r^*,V_r^*,h_r^*)} \end{cases} \tag{26}$$

Theorem 2. If model (15) has a unique solution that satisfies $d_w(c_1'p_{1C_r}' - c_r'p_{rC_r}')<1$ and $d_w(p_{rV_r}' - p_{1V_r}')<1$, then whatever κ and η are any value in $[0, 1)$, the unique solution of model (15) is asymptotically stable.

Proof. We first introduce the stability theory of nonlinear system[31]: if all the eigenvalues of the Jacobian matrix at the equilibrium point have moduli of less than one, then the equilibrium point is asymptotically stable. We call the unique solution of model (15) as the equilibrium point. Suppose that J is the Jacobian matrix of model (15), it can be represented as

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \tag{27}$$

where

$$J_{11} = \begin{pmatrix} \dfrac{\partial C_1(n)}{\partial C_1(n-1)} & \cdots & \dfrac{\partial C_1(n)}{\partial C_m(n-1)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial C_m(n)}{\partial C_1(n-1)} & \cdots & \dfrac{\partial C_m(n)}{\partial C_m(n-1)} \end{pmatrix} = \begin{pmatrix} \kappa + d_w(1-\kappa)c_1'p_{1C_1}' & \cdots & d_w(1-\kappa)c_1'p_{1C_m}' \\ \vdots & \ddots & \vdots \\ d_w(1-\kappa)c_m'p_{mC_1}' & \cdots & \kappa + d_w(1-\kappa)c_m'p_{mC_m}' \end{pmatrix} \tag{28}$$

$$J_{12} = \begin{pmatrix} \dfrac{\partial C_1(n)}{\partial V_1(n-1)} & \cdots & \dfrac{\partial C_1(n)}{\partial V_m(n-1)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial C_m(n)}{\partial V_1(n-1)} & \cdots & \dfrac{\partial C_m(n)}{\partial V_m(n-1)} \end{pmatrix} = \begin{pmatrix} d_w(1-\kappa)c_1'p_{1V_1}' & \cdots & d_w(1-\kappa)c_1'p_{1V_m}' \\ \vdots & \ddots & \vdots \\ d_w(1-\kappa)c_m'p_{mV_1}' & \cdots & d_w(1-\kappa)c_m'p_{mV_m}' \end{pmatrix} \tag{29}$$

$$J_{21} = \begin{pmatrix} \dfrac{\partial V_1(n)}{\partial C_1(n-1)} & \cdots & \dfrac{\partial V_1(n)}{\partial C_m(n-1)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial V_m(n)}{\partial C_1(n-1)} & \cdots & \dfrac{\partial V_m(n)}{\partial C_m(n-1)} \end{pmatrix} = \begin{pmatrix} -d_w(1-\eta)p_{1C_1}' & \cdots & -d_w(1-\eta)p_{1C_m}' \\ \vdots & \ddots & \vdots \\ -d_w(1-\eta)p_{mC_1}' & \cdots & -d_w(1-\eta)p_{mC_m}' \end{pmatrix} \tag{30}$$

$$J_{22} = \begin{pmatrix} \dfrac{\partial V_1(n)}{\partial V_1(n-1)} & \cdots & \dfrac{\partial V_1(n)}{\partial V_m(n-1)} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial V_m(n)}{\partial V_1(n-1)} & \cdots & \dfrac{\partial V_m(n)}{\partial V_m(n-1)} \end{pmatrix} = \begin{pmatrix} \eta - d_w(1-\eta)p_{1V_1}' & \cdots & -d_w(1-\eta)p_{1V_m}' \\ \vdots & \ddots & \vdots \\ -d_w(1-\eta)p_{mV_1}' & \cdots & \eta - d_w(1-\eta)p_{mV_m}' \end{pmatrix} \tag{31}$$

According to Eqs. (25) and (26), we have $p_{rC_1}' + \cdots + p_{rC_m}' = 0$ and $p_{rV_1}' + \cdots + p_{rV_m}' = 0$; then the characteristic polynomial of the Jacobian matrix J

can be represented as

$$\det|J-\lambda E|=$$
$$(\kappa-\lambda)[\kappa-(1-\kappa)d_w(c_1'p_{1C_2}'-c_2'p_{2C_2}')-\lambda]\cdots$$
$$[\kappa-(1-\kappa)d_w(c_1'p_{1C_m}'-c_m'p_{mC_m}')-\lambda](\eta-\lambda)\cdot$$
$$[\eta-(1-\eta)d_w(p_{2V_2}'-p_{1V_2}')-\lambda]\cdots$$
$$[\eta-(1-\eta)d_w(p_{mV_m}'-p_{1V_m}')-\lambda] \quad (32)$$

We thus know that the Jacobian matrix J has $2m$ eigenvalues $\lambda_1=\kappa$, $\lambda_2=\kappa-(1-\kappa)d_w(c_1'p_{1C_2}'-c_2'p_{2C_2}')$, $\cdots,\lambda_m=\kappa-(1-\kappa)\cdot d_w(c_1'p_{1C_m}'-c_m'p_{mC_m}'),\lambda_{m+1}=\eta$, $\lambda_{m+2}=\eta-(1-\eta)d_w(p_{2V_2}'-p_{1V_2}'),\cdots,\lambda_{2m}=\eta-(1-\eta)$ $d_w(p_{mV_m}'-p_{1V_m}')$. Because $0\leqslant\kappa<1$ and $0\leqslant\eta<1$, we know that $|\lambda_1|<1$ and $|\lambda_{m+1}|<1$. Next, we will prove that the moduli of other eigenvalues are less than one.

Suppose that $|\lambda_2|<1, \cdots, |\lambda_m|<1$, we have

$$|\lambda_r|=|\kappa-(1-\kappa)d_w(c_1'p_{1C_r}'-c_r'p_{rC_r}')|<1,$$
$$\forall r=2,\cdots,m \quad (33)$$

For further calculating, we obtain

$$\begin{cases} d_w(c_1'p_{1C_r}'-c_r'p_{rC_r}')>-\dfrac{1-\kappa}{1-\kappa}=-1 \\ d_w(c_1'p_{1C_r}'-c_r'p_{rC_r}')<1+\dfrac{2\kappa}{1-\kappa} \end{cases} \quad (34)$$

According to Eqs. (24) and (25), we know that $c_r'>0$, $p_{1C_r}'>0$ and $p_{rC_r}'<0$, so $d_w(c_1'p_{1C_r}'-c_r'p_{rC_r}')>0$. Furthermore, since $d_w(c_1'p_{1C_r}'-c_r'p_{rC_r}')<1$, we know that Ineq. (34) is set up, that is, the moduli of $\lambda_2,\cdots,\lambda_m$ are less than one. Similarly, we also know that $d_w(p_{rV_r}'-p_{1V_r}')>0$ and the moduli of $\lambda_{m+1},\cdots,\lambda_{2m}$ are less than one.

In summary, we know that all the eigenvalues of the Jacobian matrix J at the equilibrium point have moduli of less than one. Therefore, the unique solution of model (15) is asymptotically stable.

3 Model simulation

A road network with 4 OD pairs, 13 nodes, and 19 links as illustrated in Fig.1 is used to demonstrate the properties of these models established in this study.

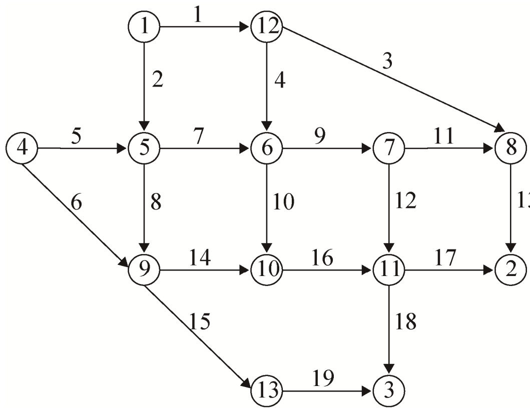

Fig. 1 Test road network

In this numerical example, assuming travel demands $d_{12}=40$, $d_{13}=80$, $d_{42}=60$, $d_{43}=20$ and link-travel time c_a is

$$c_a(f)=c_0\left[1+0.15\left(\frac{f_a}{K_a}\right)^4\right] \quad (35)$$

where c_0 is the free flow travel time on link a, and K_a is the capacity on link a. The link travel time function parameters for this network are presented in Tab.1.

Tab. 1 Parameters of link travel time function

Link No.	c_0	K_a	Link No.	c_0	K_a
1	8	70	11	1	80
2	7	100	12	3	85
3	8	30	13	6	60
4	2	75	14	3.5	85
5	2	90	15	2	50
6	6	50	16	5.5	160
7	5	140	17	4	120
8	4	90	18	1	110
9	5	120	19	8	50
10	2.5	85			

To satisfy the stability conditions, we suppose that system parameters $\kappa=0.9$, $\eta=0.9$, $\phi=0.3$, $\phi=0.3$ and $\theta=0.3$. Then we will calculate and analyze the three models under a single price regulation, single quantity regulation, and price-quantity regulation, respectively.

3.1 Single price regulation

The evolutionary trajectories of path flow and expected travel time on path 1~8 under a single price regulation are shown in Fig. 2~3. We can observe that the path flow and expected travel time have reached a steady state after a period of fluctuation. After stabilization, each path flow h_r and expected travel time C_r are listed as given in Tab. 2. The results of path flow and expected travel time on path 4~5 in Tab. 2 show that the path flow on the more expected travel time path is lower than that on the less expected travel time path under a single price regulation. This means that more travelers choose to travel on the path with less expected travel time.

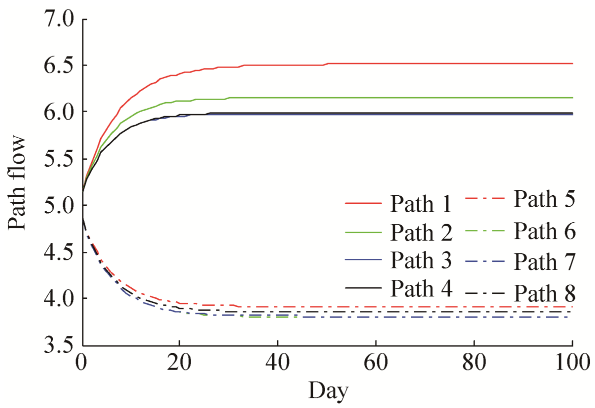

Fig. 2 Evolutionary trajectories of path flow

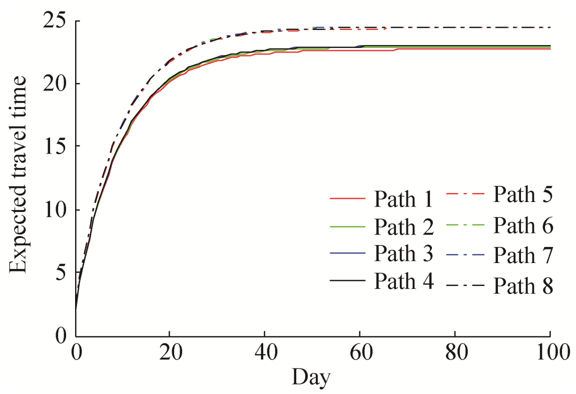

Fig. 3 Evolutionary trajectories of expected travel time

Tab. 2 Equilibrium results under single price regulation

OD Pair	Path No.	Links in Path	h_r	C_r
(1,2)	1	1,3,13	6.5108	22.6730
	2	1,4,9,11,13	6.1504	22.8628
	3	1,4,9,12,17	5.9761	22.9586
	4	1,4,10,16,17	5.9873	22.9523
	5	2,7,9,11,13	3.9123	24.3708
	6	2,7,9,12,17	3.8014	24.4666
	7	2,7,10,16,17	3.8085	24.4603
	8	2,8,14,16,17	3.8532	24.4215
(1,3)	9	1,4,9,12,18	17.3881	19.9285
	10	1,4,10,16,18	17.4209	19.9223
	11	2,7,9,12,18	11.0606	21.4365
	12	2,7,10,16,18	11.0814	21.4302
	13	2,8,14,16,18	11.2113	21.3914
	14	2,8,15,19	11.8376	21.2102
(4,2)	15	5,7,9,11,13	12.1464	19.2924
	16	5,7,9,12,17	11.8021	19.3882
	17	5,7,10,16,17	11.8244	19.3819
	18	5,8,14,16,17	11.9630	19.3431
	19	6,14,16,17	12.2642	19.2602
(4,3)	20	5,7,9,12,18	3.2144	16.3581
	21	5,7,10,16,18	3.2204	16.3518
	22	5,8,14,16,18	3.2581	16.3130
	23	5,8,15,19	3.4401	16.1318
	24	6,14,16,18	3.3402	16.2301
	25	6,15,19	3.5268	16.0489

3.2 Single quantity regulation

The evolutionary trajectories of path flow and expected residual capacity on path 1~8 under a single quantity regulation are shown in Fig. 4 and Fig. 5. We can observe that the path flow and expected residual capacity have reached a steady state after a period of fluctuation. After stabilization, each path flow h_r and expected residual capacity V_r are listed as given in Tab. 3. The results of path flow and expected residual capacity on path 4~5 in Tab. 3 show that the path flow on the more expected residual capacity path is higher than that on the less expected residual capacity path under a single quantity regulation. This means that

https://dc-china-simulation.researchcommons.org/journal/vol30/iss11/4
DOI: 10.16182/j.issn1004731x.joss.201811004

more travelers choose to travel on the path with more free driving opportunity and better driving comfort.

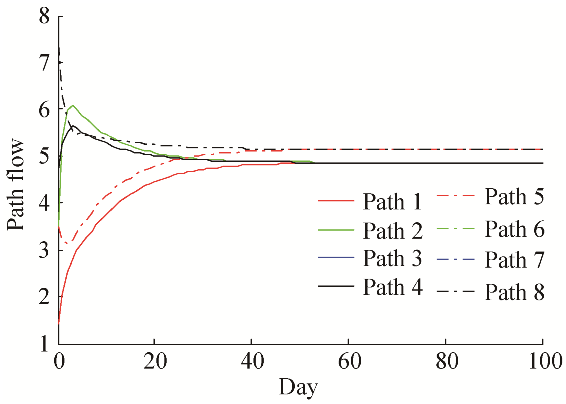

Fig. 4 Evolutionary trajectories of path flow

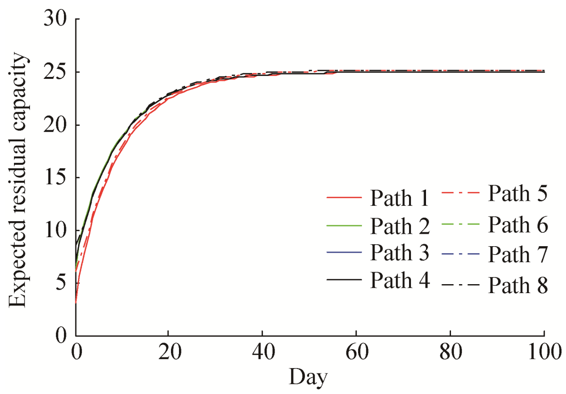

Fig. 5 Evolutionary trajectories of expected residual capacity

3.3 Price-quantity regulation

Suppose that the travelers' preference for path-travel time is 80% and for path-residual capacity is 20%, that is $\lambda=0.8$. The evolutionary trajectories of path flow and expected comprehensive travel cost on path 1~8 under a price-quantity regulation are shown in Fig. 6~7. We can observe that the path flow and expected comprehensive travel cost have reached a steady state after a period of fluctuation. After stabilization, each path flow h_r and expected comprehensive travel cost S_r are listed as given in Tab.4. The results of path flow and expected comprehensive travel cost on path 4~5 in Tab.4 show that the path flow on the more expected comprehensive travel cost path is lower than that on the less expected comprehensive travel cost path under a price-quantity regulation.

Tab. 3 Equilibrium results under single quantity regulation

OD Pair	Path No.	Links in Path	h_r	V_r
(1,2)	1	1,3,13	4.857 6	24.904 5
	2	1,4,9,11,13	4.857 7	24.904 6
	3	1,4,9,12,17	4.857 7	24.904 6
	4	1,4,10,16,17	4.857 7	24.904 6
	5	2,7,9,11,13	5.142 2	25.094 4
	6	2,7,9,12,17	5.142 3	25.094 4
	7	2,7,10,16,17	5.142 3	25.094 4
	8	2,8,14,16,17	5.142 3	25.094 4
(1,3)	9	1,4,9,12,18	12.832 1	24.904 6
	10	1,4,10,16,18	12.832 1	24.904 6
	11	2,7,9,12,18	13.584 1	25.094 4
	12	2,7,10,16,18	13.584 1	25.094 4
	13	2,8,14,16,18	13.584 1	25.094 4
	14	2,8,15,19	13.583 7	25.094 3
(4,2)	15	5,7,9,11,13	11.761 8	29.838 7
	16	5,7,9,12,17	11.762 0	29.838 7
	17	5,7,10,16,17	11.762 0	29.838 7
	18	5,8,14,16,17	11.762 0	29.838 7
	19	6,14,16,17	12.952 2	30.160 0
(4,3)	20	5,7,9,12,18	3.278 2	29.838 6
	21	5,7,10,16,18	3.278 2	29.838 6
	22	5,8,14,16,18	3.278 2	29.838 6
	23	5,8,15,19	3.278 2	29.838 6
	24	6,14,16,18	3.598 7	30.149 5
	25	6,15,19	3.288 5	29.849 1

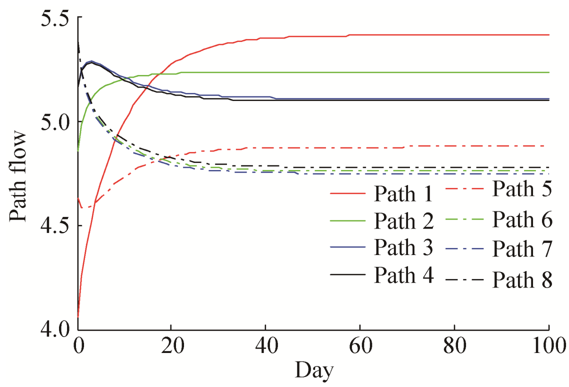

Fig. 6 Evolutionary trajectories of path flow

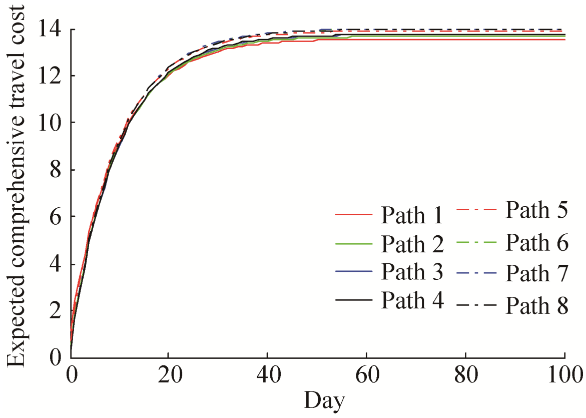

Fig. 7 Evolutionary trajectories of expected comprehensive travel cost

3.4 Comparison of equilibrium results

The network equilibrium results between OD pair (1, 2) are taken as an example. Through the comparison between path flow h_r in the 2nd column and that in the 8th column of Tab.5, we can observe that, in the changing process from the single price regulation to the price-quantity regulation, some travelers on path 1–4 have automatically transferred to path 5~8, which possesses a more expected residual capacity. Similarly, through the comparison between path flow h_r in the 5th column and that in the 8th column of Tab.5, we can find that, in the changing process from the single quantity regulation to the price-quantity regulation, some travelers on path 5~8 have automatically transferred to path 1~4, which possesses a lower expected travel time.

In summary, we can observe that the result of the price-quantity regulation is a combination of price regulation and quantity regulation, which also indicates that the day-to-day dynamical stochastic assignment model under the price-quantity regulation can simulate the influence of heterogeneous travel behaviors on the distribution characteristic and dynamic evolution of traffic flow.

Tab. 4 Equilibrium results under price-quantity regulation

OD Pair	Path No.	Links in Path	h_r	S_r
(1,2)	1	1,3,13	5.408 6	13.558 4
	2	1,4,9,11,13	5.230 5	13.670 0
	3	1,4,9,12,17	5.106 5	13.749 9
	4	1,4,10,16,17	5.094 0	13.758 1
	5	2,7,9,11,13	4.875 2	13.904 5
	6	2,7,9,12,17	4.759 6	13.984 4
	7	2,7,10,16,17	4.748 0	13.992 6
	8	2,8,14,16,17	4.777 7	13.971 8
(1,3)	9	1,4,9,12,18	13.857 3	11.319 2
	10	1,4,10,16,18	13.823 5	11.327 3
	11	2,7,9,12,18	12.915 9	11.553 7
	12	2,7,10,16,18	12.884 4	11.561 8
	13	2,8,14,16,18	12.964 9	11.541 0
	14	2,8,15,19	13.554 1	11.392 9
(4,2)	15	5,7,9,11,13	12.043 6	9.517 5
	16	5,7,9,12,17	11.758 1	9.597 5
	17	5,7,10,16,17	11.729 4	9.605 6
	18	5,8,14,16,17	11.802 7	9.584 9
	19	6,14,16,17	12.666 1	9.349 5
(4,3)	20	5,7,9,12,18	3.225 1	7.166 7
	21	5,7,10,16,18	3.217 2	7.174 9
	22	5,8,14,16,18	3.237 3	7.154 1
	23	5,8,15,19	3.383 9	7.006 5
	24	6,14,16,18	3.474 1	6.918 8
	25	6,15,19	3.462 4	6.930 1

Tab. 5 Equilibrium results of OD pair (1,2)

Path No.	Single Price Regulation			Single Quantity Regulation			Price-Quantity Regulation		
	h_r	C_r	V_r	h_r	C_r	V_r	h_r	C_r	V_r
1	6.510 8	22.673 0	10.566 8	4.857 6	22.241 8	24.904 5	5.408 6	22.317 8	21.479 2
2	6.150 4	22.862 8	10.566 9	4.857 7	22.370 6	24.904 6	5.230 5	22.457 3	21.479 3
3	5.976 1	22.958 6	10.566 9	4.857 7	22.479 3	24.904 6	5.106 5	22.557 3	21.479 3
4	5.987 3	22.952 3	10.566 9	4.857 7	22.500 3	24.904 6	5.094 0	22.567 4	21.479 3
5	3.912 3	24.370 8	31.279 6	5.142 2	24.568 8	25.094 4	4.875 2	24.510 5	28.519 7
6	3.801 4	24.466 6	31.756 5	5.142 3	24.677 5	25.094 4	4.759 6	24.610 5	28.519 8
7	3.808 5	24.460 3	31.656 4	5.142 3	24.698 5	25.094 4	4.748 0	24.620 7	28.519 8
8	3.853 2	24.421 5	39.108 8	5.142 3	24.673 5	25.094 4	4.777 7	24.594 7	28.519 8

https://dc-china-simulation.researchcommons.org/journal/vol30/iss11/4
DOI: 10.16182/j.issn1004731x.joss.201811004

4 Conclusions

Based on the disequilibrium theory in economics, we classify the travelers' route choice behavior into three types and establish three day-to-day dynamical stochastic assignment models under price regulation, quantity regulation, and price-quantity regulation, respectively. The existence, uniqueness, and stability of the solution to these presented models are proved. Through a numerical experiment, the distribution characteristic and dynamic evolution process of traffic flow under different regulation modes are analyzed. The results show that the three models of price regulation, quantity regulation, and price-quantity regulation can converge to a steady state, but there are obvious differences among the steady states of these three models. The result of price regulation is that more travelers choose to travel on the path with less expected travel time, and the result of quantity regulation is that more travelers choose to travel on the path with more free driving opportunity and better driving comfort. The result of price-quantity regulation is a combination of price regulation and quantity regulation, which changes with the setting adjustment of a weighting factor and is rather flexible considering the heterogeneity of travelers' route choice behavior.

The research on day-to-day dynamical stochastic assignment model under price-quantity regulation can help in understanding the distribution characteristic and dynamic evolution of traffic flow under the heterogeneous circumstances of travelers' route choice behavior. Moreover, it can provide new ideas for traffic control, traffic guidance, traffic information service, etc. An ongoing extension of this research seeks to appropriate quantity regulation variables according to road hierarchy, function, and service level. Another future direction is to fully consider the heterogeneity of travelers' route choice behavior in urban traffic management and control.

References:

[1] Wardrop J G. Some Theoretical Aspects of Road Traffic Research[J]. Journal of the Operational Research Society (S0160-5682), 1953, 4: 72-73.

[2] Watling D, Hazelton M L. The Dynamics and Equilibria of Day-to-Day Assignment Models[J]. Networks & Spatial Economics (S1566-113X), 2003, 3(3): 349-370.

[3] Li X F, Lang M X. Multi-Class and Multi-Criteria Stochastic User Equilibrium Model Based on Generalized Nested Logit Model[J]. Journal of Transportation Systems Engineering and Information Technology (S1570-6672), 2014, 14(4): 139-145.

[4] Smith M J, Watling D P. A Route-Swapping Dynamical System and Lyapunov Function for Stochastic User Equilibrium[J]. Transportation Research Part B: Methodological (S0191-2615), 2016, 85: 132-141.

[5] Yang J, Jiang G. Development of an Enhanced Route Choice Model Based on Cumulative Prospect Theory[J]. Transportation Research Part C: Emerging Technologies (S0968-090X), 2014, 47: 168-178.

[6] Li Q, Liao F X, Timmermans H J P, et al. A User Equilibrium Model for Combined Activity-Travel Choice under Prospect Theoretical Mechanisms of Decision-Making under Uncertainty[J]. Transportmetrica A: Transport Science (S2324-9935), 2016, 12(7): 629-649.

[7] Zhang C, Liu T L, Huang H J, et al. A Cumulative Prospect Theory Approach to Commuters' Day-to-Day Route-Choice Modeling with Friends' Travel Information [J]. Transportation Research Part C: Emerging Technologies (S0968-090X), 2018, 86: 527-548.

[8] Loomes G, Sugden R. Regret theory: an Alternative Theory of Rational Choice under Uncertainty[J]. The Economic Journal (S1468-0297), 1982, 92(368): 805-824.

[9] Chorus C G. A Generalized Random Regret Minimization Model [J]. Transportation Research Part B: Methodological (S0191-2615), 2014, 68: 224-238.

[10] Li M, Huang H J. A Regret Theory-Based Route Choice Model [J]. Transportmetrica A: Transport Science (S2324-9935), 2017, 13(3): 250-272.

[11] Peeta S, Mahmassani H S. Multiple User Classes Real-Time Traffic Assignment for Online Operations: a

Rolling Horizon Solution Framework[J]. Transportation Research Part C: Emerging Technologies (S0968-090X), 1995, 3(2): 83-98.

[12] Florian M, Wu J H, He S. A Multi-Class Multi-Mode Variable Demand Network Equilibrium Model with Hierarchical Logit Structures[J]. Journal of University of Shanghai Forence & Technology (S1007-6735), 1999, 63: 119-133.

[13] Brown D B, Giorgi E D, Sim M. Aspirational Preferences and Their Representation by Risk Measures [J]. Management Science (S0025-1909), 2012, 58(11): 2095-2113.

[14] Ramos G M, Daamen W, Hoogendoorn S. Modelling Travellers' Heterogeneous Route Choice Behaviour as Prospect Maximizers [J]. Journal of Choice Modelling (S1755-5345), 2013, 6: 17-33.

[15] Zockaie A, Saberi M, Mahmassani H S, et al. Activity-Based Model with Dynamic Traffic Assignment and Consideration of Heterogeneous User Preferences and Reliability Valuation[J]. Transportation Research Record: Journal of the Transportation Research Board (S0361-1981), 2016, 2493: 78-87.

[16] Huang Z X, He G G. Disequilibrium Transportation Planning View[C]//Traffic and Transportation Studies. Reston: ASCE, 2015: 305-310.

[17] Link H. Is Car Drivers' Response to Congestion Charging Schemes Based on the Correct Perception of Price Signals [J]. Transportation Research Part A: Policy and Practice (S0965-8564), 2015, 71: 96-109.

[18] Chiou Y C, Fu C. Responses of Drivers and Motorcyclists to Congestion Charge[J]. Transportation Research Procedia (S2352-1465), 2017, 25: 2961-2973.

[19] Huang W J, Zhang W H, Shen J Y, et al. Road Congestion Pricing Model Considering Energy Consumption Measurement [J]. Journal of Transportation Systems Engineering and Information Technology (S1570-6672), 2018, 18(1): 166-172.

[20] Habib K, Mahmoud M, Coleman J. Effect of Parking Charges at Transit Stations on Park-and-Ride Mode Choice: Lessons Learned From Stated Preference Survey in Greater Vancouver, Canada [J]. Transportation Research Record: Journal of the Transportation Research Board (S0361-1981), 2013, 2351: 163-170.

[21] Wang J J, Jin S X, Li W J, et al. Parking Pricing Method of Public Parking Lots Based on Multi-Aspect Comprehensive Benefit Maximization[J]. Journal of Traffic and Transportation Engineering (S2095-7564), 2017, 17(2): 126-135.

[22] Lin X X, Yuan P C. A Dynamic Parking Charge Optimal Control Model under Perspective of Commuters' Evolutionary Game Behavior[J]. Physica A: Statistical Mechanics and its Applications (S0378-4371), 2018, 490: 1096-1110.

[23] Hasan M, Jha M, Ben A M. Evaluation of Ramp Control Algorithms Using Microscopic Traffic Simulation[J]. Transportation Research Part C: Emerging Technologies (S0968-090X), 2002, 10(3): 229-256.

[24] Frejo J R D, Camacho E F. Global Versus Local MPC Algorithms in Freeway Traffic Control with Ramp Metering and Variable Speed Limits[J]. IEEE Transactions on Intelligent Transportation Systems (S1524-9050), 2012, 13(4): 1556-1565.

[25] Yang X F, Cheng Y, Chang G L. Integration of Adaptive Signal Control and Freeway Off-Ramp Priority Control for Commuting Corridors[J]. Transportation Research Part C: Emerging Technologies (S0968-090X), 2018, 86: 328- 345.

[26] Yao J, Hou C F, Yang Z X, et al. Optimization Study on Road Pricing with Variable Bus Frequency[J]. Journal of Transportation Systems Engineering and Information Technology (S1570-6672), 2010, 10(2): 93-99.

[27] Wang J, Hu L G, He X. Bi-Level Programming Model and the Solution Algorithm for Urban Bus Departure Frequency [J]. Optimization. Systems Engineering (S1001- 4098), 2013, 31(12): 69-73.

[28] Berrebi S J, Watkins K E, Laval J A. A Real-Time Bus Dispatching Policy to Minimize Passenger Wait on a High Frequency Route [J]. Transportation Research Part B: Methodological (S0191-2615), 2015, 81: 377-389.

[29] Ruiz M, Segui-Pons J M, Mateu-Lladó J. Improving Bus Service Levels and Social Equity Through Bus Frequency Modelling [J]. Journal of Transport Geography (S0966- 6923), 2017, 58: 220-233.

[30] Nash J. Non-Cooperative Games [J]. Annals of Mathematics (S0003-486X), 1951, 54(2): 286-295.

[31] Liu S X, Guan H Z, Yan H. Chaotic Behavior in the Dynamical Evolution of Network Traffic Flow and Its Control [J]. Acta Physica Sinica (S1000-3290), 2012, 61(9): 090506.