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Optimization of Q-BTGSID Based on Sensitivity Analysis

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Abstract: In order to make up for the defect that relative degree of incidence, absolute degree of incidence and synthetic degree of incidence are limited in the range of (0.5, 1], this paper attempts to improve the degree of grey incidence. A *control factor of "λ" and the metric space are set up to adjust. A new model is established and its specific properties are studied. It is proved that the new model satisfies the grey incidence axioms and the range of degree of grey incidence can be extended to (0, 1].* We put *forward four principles of the quasi-optimal value, and summarize the specific algorithm steps according to sensitivity analysis.* The performances of all kinds of analysis models of *degree of incidence* are compared and *the quasi-optimal value is obtained*. In addition, the paper succeeds in showing that the new model not only *keeps the original relation order*, but also extends the range of degree of grey incidence to a broader scope, and improves the resolution.

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基于灵敏度分析的 Q-BTGSID 的优化

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摘要：为了弥补相对关联度，绝对关联度和综合关联度在取值范围上存在的不足，试图做了以下改进。*引入可调因子和空间距离来调节，构建新模型，证明了改进的模型满足灰色关联公理，能够使关联度的值分布到（0，1]这一更大的区间。提出新模型的准优解所满足的 4 个原则，如：保序性原则、极差最大化原则、对称性原则以及数值分布区间个数之和最大化原则，并根据灵敏性分析理论总结了具体算法。*结合实例，*对各类关联度分析模型在性能上进行比较，进而验证了新模型的*合理性和实用性。

关键词：灵敏性分析；综合关联度；算法；优化

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Introduction

Grey incidence analysis is one of the significant tools of system analysis, which is widely applied to the prediction, decision making, pattern recognition, control, and other fields. Professor Deng founded the grey system theory and proposed the concept of point incidence coefficients[1]. Subsequently, many kinds of models were established in different approach. Liu, et al. summarized absolute degree of incidence and relative degree of incidence[2]. Zhang et al. extended

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the application of absolute incidence degree to interval grey number[3]. Liu et al. constructed nearness degree of incidence and similarity degree of incidence, and summarized the research progress of grey incidence analysis model[4-5]. Zhang and Liu proposed three dimensional degree of grey incidence analysis model[6]. Yu et al. used the maximum entropy principle and established the model of maximum entropy[7]. Jiang et al. studied the grey relational decision model[8], and so on.

In application, Fang et al. put forward the comprehensive evaluation model of grey entropy, and applied it to the decision-making method of intercity rail transit system[9]. Guan and Song used the grey incidence analysis theory and applied it to multi-sensor data fusion method[10]. Li et al. used the degree of grey incidence to determine the pull factors, and structured the evaluation model of regional coordinated development system[11]. Zhang et al. established an investment evaluation model and applied it to the national grid system analysis[12], and so on.

In conclusion, at present, the research on grey incidence analysis is roughly divided into two classes, one kind is qualitative analysis: the main judge relation order, the other kind is quantitative calculation for some relevance parameter. However, certain models have some drawbacks, and the following are the typical ones. Firstly, values of the absolute degree of incidence, relative degree of incidence and synthetic degree of incidence are greater than 0.5 and asymmetry, and the distinguished effect is not obvious. Secondly, in quantitative analysis, such as Li et al.[11], in the study of the coordinated development of regional economy, stimulating factors of specific values will affect the development degree and comprehensive coordinated development degree of the calculation results.

In order to solve those problems, the rest of the paper is organized as follows. The limitations of absolute degree of incidence, relative degree of incidence and synthetic degree of incidence are proved in Section 1. In Section 2, first of all, we propose a new model to overcome the leak of the original model and study their properties. Secondly, we also put forward some principles of the quasi-optimal value, such as the principle of consistency, the principle of range analysis maximization and the principle of symmetry, and so on. Thirdly, we optimize the value of the new model based on the theory of sensitivity analysis. In Section 3, we compare each kind of value of the degree of incidence through example, and obtain some useful results. Finally, conclusions are given in Section 4.

1 Definitions and limitations

1.1 Basic concepts

Definition 1[2] Liu et al. assume that the images of zero starting point have two behavioral sequences

$X_i = (x_i(1), x_i(2), \cdots, x_i(n))$,
$X_j = (x_j(1), x_j(2), \cdots, x_j(n))$

and these two sequences have the same length,

$X_i^0 = (x_i^0(1), x_i^0(2), \cdots, x_i^0(n))$,
$X_j^0 = (x_j^0(1), x_j^0(2), \cdots, x_j^0(n))$

then：

$$\varepsilon_{ij} = \frac{1+|s_i|+|s_j|}{1+|s_i|+|s_j|+|s_i - s_j|} \tag{1}$$

is called the absolute degree of incidence (GAID) of X_i and X_j, where

$s_i = \int_1^n (X_i - x_i(1))dt, s_j =$
$\int_1^n (X_j - x_j(1))dt, s_i - s_j = \int_1^n (X_i^0 - X_j^0)dt$,
$x_i^0(k) = x_i(k) - x_i(1)$, $x_j^0(k) = x_j(k) - x_j(1)$,
$(k = 1, 2, \cdots, n)$.

Assume that X_i and X_j are two sequences of the same

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length with non-zero initial values, X'_i and X'_j are the initial images of X_i and X_j, the images of zero starting point of the two behavioral sequences X'_i, X'_j are X'^0_i and X'^0_j. The absolute degree of incidence (GAID) of X'_i and X'_j is called the relative degree of incidence (GRID) of X_i and X_j. Denote,

$$r_{ij} = \frac{1+|s'_i|+|s'_j|}{1+|s'_i|+|s'_j|+|s'_i - s'_j|} \quad , \tag{2}$$

then

$$\rho_{ij} = \theta\varepsilon_{ij} + (1-\theta) r_{ij} \tag{3}$$

is called the synthetic degree of incidence (GSID) of X_i and X_j, where $\theta \in [0,1]$.

Lemma 1 [2] Assume that X_i and X_j are 1-time -interval sequences of the same length, and

$$X_i^0 = (x_i^0(1), x_i^0(2), \cdots, x_i^0(n)),$$
$$X_j^0 = (x_j^0(1), x_j^0(2), \cdots, x_j^0(n))$$

are the zero images of X_i and X_j, then:

$$|s_i| = \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2} x_i^0(n)\right|,$$

$$|s_j| = \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2} x_j^0(n)\right|,$$

$$|s_i - s_j| = \left|\sum_{k=2}^{n-1} [x_i^0(k) - x_j^0(k)] + \frac{1}{2}[x_i^0(n) - x_j^0(n)]\right|. \tag{4}$$

1.2 The limitations of GSID

It is obvious that $\varepsilon_{ij} \in (0.5,1]$, $r_{ij} \in (0.5,1]$, and $\theta \in [0,1]$,

(1)When $\varepsilon_{ij} = r_{ij}$, $\rho_{ij} = \varepsilon_{ij} = r_{ij}$;

(2)When $\varepsilon_{ij} > r_{ij}$,

$$r_{ij} \leqslant \theta(\varepsilon_{ij} - r_{ij}) + r_{ij} \leqslant (\varepsilon_{ij} - r_{ij}) + r_{ij} = \varepsilon_{ij}$$

and

$$\rho_{ij} = \theta\varepsilon_{ij} + (1-\theta) r_{ij} = \theta(\varepsilon_{ij} - r_{ij}) + r_{ij},$$

so $\rho_{ij} \in [r_{ij}, \varepsilon_{ij}] \subsetneq (0.5,1]$.

(3)Where $\varepsilon_{ij} < r_{ij}$, we have $\rho_{ij} \in (0.5,1]$, especial, $\rho_{ii} = 1$;

Hence, $\rho_{ij} \in (0.5,1]$.

Therefore, that scope is limited within the range of $(0.5,1]$. So, can its distribution area be expanded to a wider range through the improvement?

2 Model and method

2.1 The establishment of the new model

The model of absolute degree of incidence considers the relationship between absolute amount of sequences. The model of relative degree of incidence considers the relative to the starting point of the relationship between the rate of change. On this basis, we consider the metric space between the two sequences. Under normal circumstances, the smaller metric space leads to the closer relationship between sequences, the greater degree of grey incidence, on the contrary, results in smaller. To this end, we proposed the definition as follows.

Definition 2 Assume that two sequences X_i and X_j are 1-time-interval of the same length, other conditions are same to definition 1, then,

$$\varepsilon_{ij}^{(\lambda,p)} = \frac{1+|s_i|+|s_j|}{1+|s_i|+|s_j|+|s_i - s_j| + \lambda \cdot d_{ij}^p} \tag{5}$$

and

$$r_{ij}^{(\lambda,p)} = \frac{1+|s'_i|+|s'_j|}{1+|s'_i|+|s'_j|+|s'_i - s'_j| + \lambda \cdot d'^p_{ij}} \tag{6}$$

are respectively called the quasi-absolute degree of incidence (Q-GAID) and the quasi-relative degree of incidence (Q-GRID) of X_i and X_j, where $\lambda \in [0,1]$,

$$d_{ij}^p = (\sum_{k=1}^{n} \left|x_i^0(k) - x_j^0(k)\right|^p)^{\frac{1}{p}},$$

$$d'^p_{ij} = (\sum_{k=1}^{n} \left|x'_i(k) - x'_j(k)\right|^p)^{\frac{1}{p}}. \tag{7}$$

Then

$$\rho_{ij}^{(\lambda,p)} = \sigma_1^{(\lambda,p)} \varepsilon_{ij}^{(\lambda,p)} + \sigma_2^{(\lambda,p)} r_{ij}^{(\lambda,p)} \tag{8}$$

is called the quasi-synthetic degree of incidence (Q-GSID) of X_i and X_j, where,

$$\sigma_1^{(\lambda,p)} > 0, \sigma_2^{(\lambda,p)} > 0, \sigma_1^{(\lambda,p)} + \sigma_2^{(\lambda,p)} = 1. \tag{9}$$

All of the three models are called the

quasi-degree of incidence (Q-GID).

Especial:

(1)When $\lambda = 0$, the quasi-degree of incidence (Q-GID) is the same to definition1;

(2)When $\lambda = 1, p = 2$, it is called the quasi-"BT"- degree of incidence (Q-BTGID), then

$$\varepsilon_{ij}^{(1,2)} = \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right| + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right| + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n)\right| + \left|\sum_{k=2}^{n-1} [x_i^0(k) - x_j^0(k)] + \frac{1}{2}[x_i^0(n) - x_j^0(n)]\right| + \left(\sum_{k=1}^{n} \left|x_i^0(k) - x_j^0(k)\right|^2\right)^{\frac{1}{2}}\right]^{-1} \tag{10}$$

and

$$r_{ij}^{(1,2)} = \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2}x'^0_i(n)\right| + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2}x'^0_j(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2}x'^0_i(n)\right| + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2}x'^0_j(n)\right| + \left|\sum_{k=2}^{n-1} [x'^0_i(k) - x'^0_j(k)] + \frac{1}{2}[x'^0_i(n) - x'^0_j(n)]\right| + \left(\sum_{k=1}^{n} \left|x'_i(k) - x'_j(k)\right|^2\right)^{\frac{1}{2}}\right]^{-1} \tag{11}$$

are respectively called the quasi-"BT"-absolute degree of incidence (Q-BTGAID) and the quasi-"BT"-relative degree of incidence (Q-BTGRID). Then

$$\rho_{ij}^{(1,2)} = \sigma_1^{(1,2)}\varepsilon_{ij}^{(1,2)} + \sigma_2^{(1,2)} r_{ij}^{(1,2)} \tag{12}$$

is called the quasi-"BT"-synthetic degree of incidence (Q-BTGSID)of X_i and X_j, where

$$\sigma_1^{(1,2)} > 0, \sigma_2^{(1,2)} > 0, \sigma_1^{(1,2)} + \sigma_2^{(1,2)} = 1. \tag{13}$$

(3)When $\lambda = \frac{1}{\sqrt{n}}, p = 2$, it is called the quasi-"Euclidean"-degree of incidence (Q-EUGID),then

$$\varepsilon_{ij}^{(\frac{1}{\sqrt{n}},2)} = \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right| + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right| + \frac{1}{\sqrt{n}} \cdot \left(\sum_{k=1}^{n} \left|x_i^0(k) - x_j^0(k)\right|^2\right)^{\frac{1}{2}} + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n)\right| + \left|\sum_{k=2}^{n-1} [x_i^0(k) - x_j^0(k)] + \frac{1}{2}[x_i^0(n) - x_j^0(n)]\right|\right]^{-1} \tag{14}$$

and

$$r_{ij}^{(\frac{1}{\sqrt{n}},2)} = \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2}x'^0_i(n)\right| + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2}x'^0_j(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2}x'^0_i(n)\right| + \frac{1}{\sqrt{n}} \cdot \left(\sum_{k=1}^{n} \left|x'_i(k) - x'_j(k)\right|^2\right)^{\frac{1}{2}} + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2}x'^0_j(n)\right| + \left|\sum_{k=2}^{n-1} [x'^0_i(k) - x'^0_j(k)] + \frac{1}{2}[x'^0_i(n) - x'^0_j(n)]\right|\right]^{-1} \tag{15}$$

are respectively called the quasi-"Euclidean"-absolute degree of incidence (Q-EUGAID) and the quasi-"Euclidean"-relative degree of incidence (Q-EUGRID). Then

$$\rho_{ij}^{(\frac{1}{\sqrt{n}},2)} = \sigma_1^{(\frac{1}{\sqrt{n}},2)} \varepsilon_{ij}^{(\frac{1}{\sqrt{n}},2)} + \sigma_2^{(\frac{1}{\sqrt{n}},2)} r_{ij}^{(\frac{1}{\sqrt{n}},2)} \tag{16}$$

is called the quasi-"Euclidean"-synthetic degree of

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incidence (Q-EUGSID) of X_i and X_j, where

$$\sigma_1^{(\frac{1}{\sqrt{n}},2)} > 0, \sigma_2^{(\frac{1}{\sqrt{n}},2)} > 0, \sigma_1^{(\frac{1}{\sqrt{n}},2)} + \sigma_2^{(\frac{1}{\sqrt{n}},2)} = 1 \quad (17).$$

(4)When $\lambda = 1, p = 1$, it is called the quasi-“very close”-degree of incidence (Q-MSGID), then

$$\varepsilon_{ij}^{(1,1)} = \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2} x_i^0(n)\right| + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2} x_j^0(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2} x_i^0(n)\right| + \sum_{k=1}^{n} \left|x_i^0(k) - x_j^0(k)\right| + \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2} x_j^0(n)\right| + \left|\sum_{k=2}^{n-1}[x_i^0(k) - x_j^0(k)] + \frac{1}{2}[x_i^0(n) - x_j^0(n)]\right|\right]^{-1} \quad (18)$$

and

$$r_{ij}^{(1,1)} = \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2} x'^0_i(n)\right| + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2} x'^0_j(n)\right|\right] \times \left[1 + \left|\sum_{k=2}^{n-1} x'^0_i(k) + \frac{1}{2} x'^0_i(n)\right| + \sum_{k=1}^{n} \left|x'_i(k) - x'_j(k)\right| + \left|\sum_{k=2}^{n-1} x'^0_j(k) + \frac{1}{2} x'^0_j(n)\right| + \left|\sum_{k=2}^{n-1}[x'^0_i(k) - x'^0_j(k)] + \frac{1}{2}[x'^0_i(n) - x'^0_j(n)]\right|\right]^{-1} \quad (19)$$

are respectively called quasi - “very close”-absolute degree of incidence(Q-MSGAID)and quasi-“very close”-relative degree of incidence (Q-MSGRID). Then

$$\rho_{ij}^{(1,1)} = \sigma_1^{(1,1)} \varepsilon_{ij}^{(1,1)} + \sigma_2^{(1,1)} r_{ij}^{(1,1)} \quad (20)$$

is called the quasi-“very close”-synthetic degree of incidence (Q-MSGSID) of X_i and X_j, where

$$\sigma_1^{(1,1)} > 0, \sigma_2^{(1,1)} > 0, \sigma_1^{(1,1)} + \sigma_2^{(1,1)} = 1. \quad (21)$$

2.2 The properties of Q-GID

Theorem 1 The quasi-degree of incidence

$$\varepsilon_{ij}^{(\lambda,p)} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j| + \lambda \cdot d_{ij}^p}$$

and $$r_{ij}^{(\lambda,p)} = \frac{1 + |s'_i| + |s'_j|}{1 + |s'_i| + |s'_j| + |s'_i - s'_j| + \lambda \cdot d'^p_{ij}}$$

all satisfy the properties of normality and closeness.

Proof. (1) The properties of normality.

It is obvious that $\varepsilon_{ij}^{(\lambda,p)} > 0$, and $|s_i - s_j| \geqslant 0, d_{ij}^p \geqslant 0$, hence, $0 < \varepsilon_{ij}^{(\lambda,p)} \leqslant 1, \varepsilon_{ij}^{(\lambda,p)} = 1 \Leftarrow X_i = X_j$.

(2) The properties of closeness.

When $\lambda \neq 0$, the smaller $|x_i(k) - x_j(k)|$ is, the larger ε_{ij} will be, the smaller $\lambda \cdot d_{ij}^p$, the larger $\varepsilon_{ij}^{(\lambda,p)}$. When $\lambda = 0$, it is obvious.

Similarly, Q-GRID’ property can be proven.

Theorem 2 The range of Q-GAID, Q-GRID and Q-GSID can extend to (0, 1].

Proof. When $X_i \neq X_j$ and $\lambda \neq 0$,

$$\frac{|s_i - s_j| + \frac{1}{\sqrt{n}} \cdot d_{ij}^2}{1 + |s_i| + |s_j|} = \frac{|s_i - s_j| + \lambda \cdot (\sum_{k=1}^{n} |x_i^0(k) - x_j^0(k)|^p)^{\frac{1}{p}}}{1 + |s_i| + |s_j|} > 0.$$

Hence $1 + \frac{|s_i - s_j| + \lambda \cdot d_{ij}^p}{1 + |s_i| + |s_j|} > 1$.

So

$$\varepsilon_{ij}^{(\lambda,p)} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j| + \lambda \cdot d_{ij}^p} = \frac{1}{1 + \frac{|s_i - s_j| + \lambda \cdot d_{ij}^p}{1 + |s_i| + |s_j|}} < 1.$$

When $X_i = X_j$, $\varepsilon_{ij}^{(\lambda,p)} = 1$, Then $\varepsilon_{ij}^{(\lambda,p)} \in (0,1]$, Similarly $r_{ij}^{(\lambda,p)} \in (0,1]$, $\rho_{ij}^{(\lambda,p)} \in (0,1]$.

At the same time, we can summarize the specific steps of Q-GSID as follows: here we only introduce the case of Q-BTGSID, similarly, we can summarize the others.

2.3 Optimization algorithm of Q-BTGSID based on sensitivity analysis

2.3.1 The principle of the Q-BTGSID

(1) Principle 1: The principle of consistency

It shows that the results of the analysis are in line with the actual situation if relation order of the improved model is the same as the original model, and have a certain practicality and feasibility.

(2) Principle 2：The principle of range analysis maximization

In grey incidence analysis, normally, the greater the range between data centers and the higher the degree of differentiation, the better the distinguish effect.

(3) Principle 3：The principle of symmetry

The scope of Q-BTGSID is extended from (0.5,1] to(0,1], which not only improves the resolution, but also reveals symmetry. Hence, it overcomes the defect of the original model.

(4) Principle 4：The principle of *SUM* maximization

The sum of different interval number is labeled as *SUM*, and interval length is set to 0.1. When each range is equal or very close, we can consider the *SUM*. The greater the *SUM* is, the better Q-BTGSID will be.

2.3.2 Algorithm steps of the Q-BTGSID

Step1. Transform original data into 1-time-interval sequence.

Assume that the length of X_i is smaller than the length of X_j.

Firstly, the operator of D_1 is used in order to transform X_i into a sequence with the same time interval as X_i. Let

$$x_i(\xi)d_1 = \frac{x_i(\xi_{n_1}) + x_i(\xi_{n_2})}{2}$$

where ξ_{n1} and ξ_{n2} are odd number, orξ_{n1} and ξ_{n2} are even number, and

$$\xi = \frac{\xi_{n_1} + \xi_{n_2}}{2}, n_1 \in N^+, n_2 \in N^+,$$
$$1 \leqslant \xi_{n_1} < \xi_{n_2} \leqslant n, (\xi = 2,3,\cdots,n-1).$$

Secondly, transform X_i and X_j into 1-time-interval sequence

$$X_i = (x_i(1), x_i(2), \cdots, x_i(n)),$$
$$X_j = (x_j(1), x_j(2), \cdots, x_j(n)).$$

Where

$$x_i(\xi) = \frac{x_i(\xi-1) + x_i(\xi+1)}{2},$$
$$x_j(\xi) = \frac{x_j(\xi-1) + x_j(\xi+1)}{2}.$$

Step2. Introduce a control factor of "λ" and the metric space of d_{ij}^p and d'^{p}_{ij}, where

$$d_{ij}^p = (\sum_{k=1}^{n} \left| x_i^0(k) - x_j^0(k) \right|^p)^{\frac{1}{p}},$$

$$d'^{p}_{ij} = (\sum_{k=1}^{n} \left| x'_i(k) - x'_j(k) \right|^p)^{\frac{1}{p}}.$$

Step3. Calculate the value of Q-BTGAID and Q-BTGRID.

Let λ=1, p=2. The initial operator of D_2 is used to calculate the initial images of X_i and X_j

$$X'_i = X_i D_2 = (x_i(1)d_2, x_i(2)d_2, \cdots, x_i(n)d_2) = (\frac{x_i(1)}{x_i(1)}, \frac{x_i(2)}{x_i(1)}, \ldots, \frac{x_i(n)}{x_i(1)}),$$

$$X'_j = X_j D_2 = (x_j(1)d_2, x_j(2)d_2, \cdots, x_j(n)d_2) = (\frac{x_j(1)}{x_j(1)}, \frac{x_j(2)}{x_j(1)}, \cdots, \frac{x_j(n)}{x_j(1)}).$$

Secondly, the metric space of d_{ij}^2 and d'^{2}_{ij} is calculated

$$d_{ij}^2 = (\sum_{k=1}^{n} \left| x_i(k) - x_j(k) \right|^2)^{\frac{1}{2}},$$

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$$d_{ij}'^{2} = (\sum_{k=1}^{n}\left|x_i'(k) - x_j'(k)\right|^2)^{\frac{1}{2}}.$$

Finally, $\varepsilon_{ij}^{(1,2)}$ and $r_{ij}^{(1,2)}$ are obtained.

Step4. Calculate the value of Q-BTGSID

We can gain the formula of Q-BTGSID

$$\rho_{ij}^{(1,2)} = \sigma_1^{(1,2)}\varepsilon_{ij}^{(1,2)} + \sigma_2^{(1,2)}r_{ij}^{(1,2)}$$

Step5. Complete sensitivity analysis

In the premise that $\sigma_1^{(1,2)} > 0$, $\sigma_2^{(1,2)} > 0$, $\sigma_1^{(1,2)} + \sigma_2^{(1,2)} = 1$ are satisfied, different values of $\sigma_1^{(1,2)}$ are given, and the values of $\rho_{ij}^{(1,2)}$ are calculated, we will gain several groups of different values.

Step6. Find the quasi-optimal value of Q-BTGSID

The relation order, SUM and range are calculated in each set of values. On this basis, we can determine the quasi-optimal value of Q-BTGSID.

3 Application

This data comes from "Statistical Yearbook of Chinese 2014". We arrange each form data in sequences. We introduce the added value of Gross Domestic Product (GDP); the added value of Farming, Forestry, Fishery, Animal Husbandry, and Water Conservancy; the added value of Industry; the added value of Construction; the added value of Wholesale and Retail; the added value of Traffic, Transport, Storage and Post; the added value of Accommodation and Restaurants; the added value of Financial Industry; the added value of real estate. They are respectively denoted as the X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, which are shown in Tab.1.

3.1 Calculation results

The value of GAID, GRID and GSID are calculated respectively. For convenience, all of the three values are recorded as α_{0i}. Let $\theta = 0.5$, the results are shown in Tab.2 and Fig.1, where $i = 1, 2, 3, \cdots, 8$.

Secondly, the value of Q-BTGAID and Q-BTGRID are computed and recorded as β_{0i}, the results are shown in Tab. 3.

Tab. 1 Added value of GDP Units：100 million yuan

Year	2007	2008	2009	2010	2011	2012	2013
X_0	268 019.4	316 751.7	345 629.2	40 8903	484 123.5	534 123	588 018.8
X_1	28 618.6	33 692.7	35 215.3	40 521.8	47 472.9	52 358.8	56 966
X_2	110 253.9	129 929.1	13 5849	162 376.4	191 570.8	204 539.5	217 263.9
X_3	15 296.5	18 743.2	22 601.1	27 177.6	32 840	36 804.8	40 807.3
X_4	20 937.8	26 182.3	29 001.5	35 904.4	43 730.5	4 9831	56 284.1
X_5	14 601	16 362.5	16 516.1	18 777	21 834.1	23 754.7	26 036.3
X_6	5 548.1	6 616.1	6957	7 712	8 565.4	9 536.9	10 228.3
X_7	15 173.3	18 312.9	21 797.4	25 679.7	30 678.2	35 187.7	41 190.5
X_8	13 809.7	14 738.7	18 966.9	23 569.9	28 167.6	31 248.3	35 987.6

Tab. 2 Degree of grey incidence

value	α_{01}	α_{02}	α_{03}	α_{04}	α_{05}	α_{06}	α_{07}	α_{08}
CAID	0.544 2	0.679 5	0.540 9	0.553 7	0.516 5	0.507 7	0.537 8	0.532 3
CRID	0.924 8	0.944 5	0.862 7	0.877 0	0.827 7	0.888 0	0.886 7	0.909 5
CSID	0.734 5	0.812 0	0.701 8	0.715 3	0.672 1	0.697 8	0.712 2	0.720 9

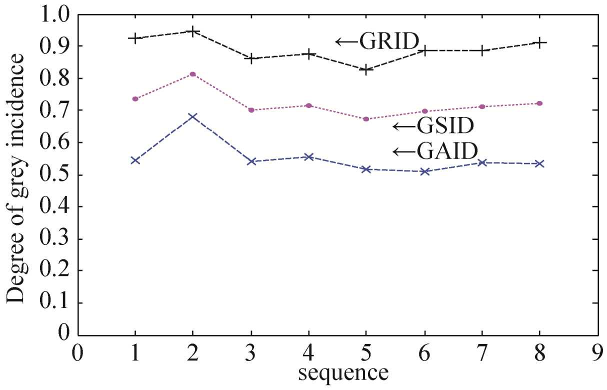

Fig. 1 GAID、GRID and GSID

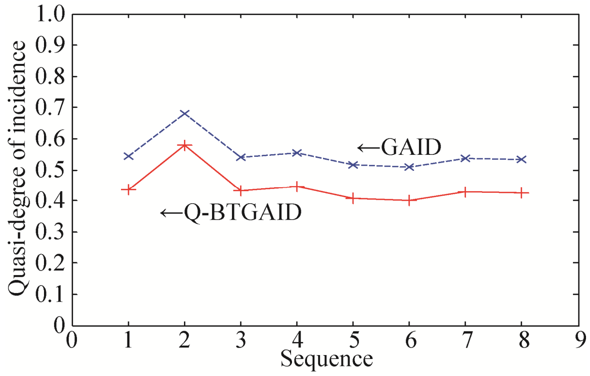

Fig. 2 GAID and Q-BTGAID

Tab. 3 Q-BTGAID and Q-BTGRID

value	β_{01}	β_{02}	β_{03}	β_{04}
Q-BTGAID	0.435 4	0.577 0	0.432 1	0.445 0
Q-BTGRID	0.887 8	0.911 5	0.804 1	0.819 6
value	β_{05}	β_{06}	β_{07}	β_{08}
Q-BTGAID	0.408 4	0.399 8	0.429 3	0.424 0
Q-BTGRID	0.759 7	0.830 9	0.829 9	0.856 3

Thirdly, GAID with Q-BTGAID are compared and some results are shown in Tab. 4 and Fig.2. Similarly, combined with Fig.3 and Tab.5, GRID and Q-BTGRID are compared.

Tab. 4 GAID and Q-BTGAID

Type	Interval	Range	Relation order
CAID	(0.5,1]	0.171 8	$X_2 \succ X_4 \succ X_1 \succ X_3 \succ X_7 \succ X_8 \succ X_5 \succ X_6$
Q-BTGAID	(0,1]	0.177 2	$X_2 \succ X_4 \succ X_1 \succ X_3 \succ X_7 \succ X_8 \succ X_5 \succ X_6$

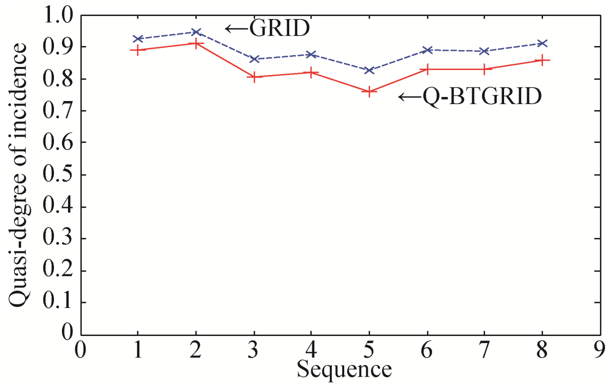

Fig. 3 GRID and Q-BTGRID

Tab.5 GRID and Q-BTGRID

Type	Interval	Range	Relation order
CRID	(0.5,1]	0.116 8	$X_2 \succ X_1 \succ X_8 \succ X_6 \succ X_7 \succ X_4 \succ X_3 \succ X_5$
Q-BTGRID	(0,1]	0.151 8	$X_2 \succ X_1 \succ X_8 \succ X_6 \succ X_7 \succ X_4 \succ X_3 \succ X_5$

Finally, to complete sensitivity analysis, we will gain some different values of Q-BTGSID, the results are seen in Tab. 6, Fig.4~ Fig.8.

Tab. 6 Different values of Q-BTGSID

$(\sigma_1^{(1,2)},\sigma_2^{(1,2)})$	$\rho_{01}^{(1,2)}$	$\rho_{02}^{(1,2)}$	$\rho_{03}^{(1,2)}$	$\rho_{04}^{(1,2)}$	$\rho_{05}^{(1,2)}$	$\rho_{06}^{(1,2)}$	$\rho_{07}^{(1,2)}$	$\rho_{08}^{(1,2)}$	Range
(0.9, 0.1)	0.4806	0.6105	0.4693	0.4825	0.4435	0.4429	0.4694	0.4672	0.1676
(0.8, 0.2)	0.5259	0.6439	0.5065	0.5199	0.4787	0.4860	0.5094	0.5105	0.1652
(0.7, 0.3)	0.5711	0.6774	0.5437	0.5574	0.5138	0.5291	0.5495	0.5537	0.1637
(0.6, 0.4)	0.6164	0.7108	0.5809	0.5948	0.5489	0.5722	0.5895	0.5969	0.1619
(0.5, 0.5)	0.6616	0.7443	0.6181	0.6373	0.5841	0.6154	0.6296	0.6402	0.1602
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮

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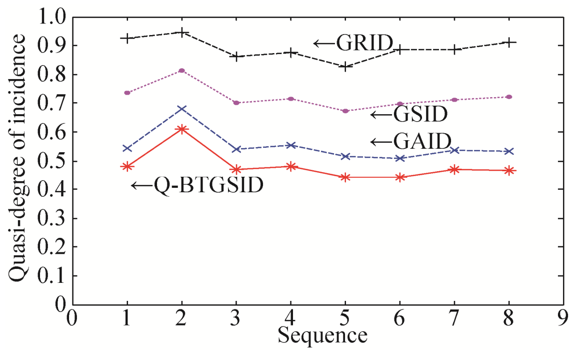

Fig. 4 $(\sigma_1^{(1,2)}, \sigma_2^{(1,2)}) = (0.9, 0.1)$

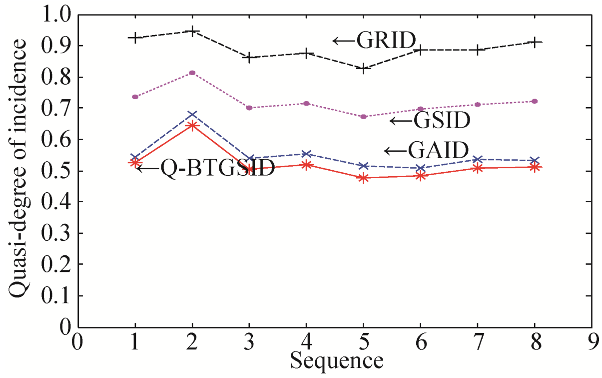

Fig. 5 $(\sigma_1^{(1,2)}, \sigma_2^{(1,2)}) = (0.8, 0.2)$

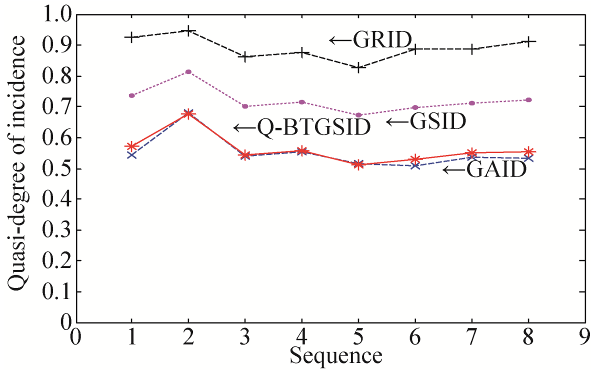

Fig. 6 $(\sigma_1^{(1,2)}, \sigma_2^{(1,2)}) = (0.7, 0.3)$

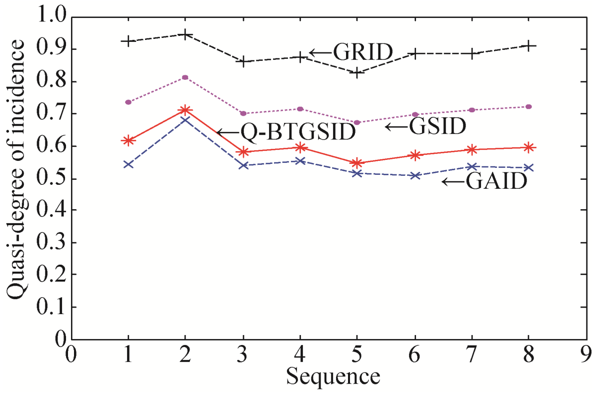

Fig. 7 $(\sigma_1^{(1,2)}, \sigma_2^{(1,2)}) = (0.6, 0.4)$

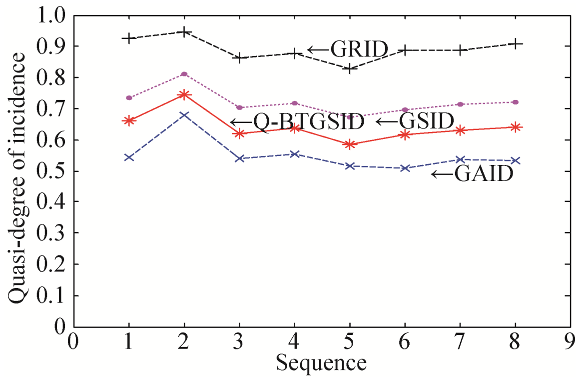

Fig. 8 $(\sigma_1^{(1,2)}, \sigma_2^{(1,2)}) = (0.5, 0.5)$

3.2 Optimization of Q-BTGSID based on sensitivity analysis

(1) From Tab. 2 and Fig.1, we can clearly see that *GAID, GRID* and *GSID* are greater than 0.5. The effectiveness of the new model is verified by this example.

(2) By comparing *Q-BTGAID* with *GAID* in Fig.2 and Tab. 4, *Q-BTGAID* would show a larger range and a wider scope of degree of grey incidence, and their relation order is

$$X_2 \succ X_4 \succ X_1 \succ X_3 \succ X_7 \succ X_8 \succ X_5 \succ X_6 .$$

The values of *Q-BTGAID* is in the range of (0.3, 0.4), (0.4, 0.5) and (0.5, 0.6), but *GAID* is only distributed in the scope of (0.5, 0.6) and (0.6, 0.7). Therefore, *Q-BTGAID* can not only satisfy the principle of consistency, but also conform to the principle of range analysis maximization, symmetry and *SUM* maximization. In the same way, we will obtain some similar results of the *Q-BTGRID* in Fig.3 and Tab. 5.

(3) Fig.4 to 8, Tab. 3 and 6 show that *Q-BTGRID* is no longer limited in the range of (0.5, 1], which can be extend to (0, 1]. They make up for the defect of the traditional model. It shows that the new model has a certain rationality and feasibility.

(4) The range of *Q-BTGAID* is increased from

0.1718 to 0.1772. As seen in each group of range, although the value of *Q-BTGSID* is reduced from 0.1676 to 0.0535, they have different degree of growth compared with *GSID*. To some extent, this will improve the resolution.

(5) The range of Fig.4 and 5 are very close, which are 0.1676 and 0.1652 respectively. Meanwhile, the values of Fig. 5 can distribute the range of (0.4, 0.5), (0.5, 0.6) and (0.6, 0.7), but the values of Fig. 4 are limited in the scope of (0.4, 0.5) and (0.5, 0.6). Hence, the degree of differentiation of Fig. 5 is better than that of Fig. 4.

We can, therefore, come to the conclusion that the quasi-optimal value of *Q-BTGSID* corresponds to the Fig.5

3.3 Performance comparison and analysis

(1) In Tab.7, we indeed observe that the value of *GAID* is always close to 0.5 and the distinguished effect is not obvious. Similarly, there are many limitations to *GRID* and *GSID*. However, the value of *Q-BTGSID* not only is extended to the range of (0, 1], but also can satisfy the principle of symmetry, which reflects the differences of relationship between each factor and *GDP better*.

(2) From Fig. 4 to 8, we can find that the range decreases with the increase of $\sigma_2^{(1,2)}$. In each model, the range and *SUM* of the quasi-optimal value of *Q-BTGSID* respectively are 0.6152 and 3. *Q-BTGSID* shows a better distinguish effect and a higher resolution compared with *GAID, GRID and GSID*.

Tab. 7 Performance comparison of GAID, GRID, GSID and Q-BTGSID

Type	Interval	Range	SUM
GAID	(0.5,1]	0.171 8	2
GRID	(0.5,1]	0.116 8	2
GSID	(0.5,1]	0.139 9	3
Q-BTGSID	(0,1]	0.165 2	3

(3) In the aspect of some grey cluster analysis, such as [2], due to *GAID, GRID* and *GSID* are all larger than 0.5, the value of *r* is always greater than 0.5. In general, it must be greater than 0.5. In addition, in some quantitative calculation, such as [11], their values are all bigger than 0.5, which directly affects the precision of some parameters. Q-BTGSID can overcome these defects.

3.4 Policy suggestions

(1) From part 3.2, it follows that the quasi-optimal value respectively are

$$\rho_{01}^{(1,2)} = 0.5259, \rho_{02}^{(1,2)} = 0.643\,9,$$
$$\rho_{03}^{(1,2)} = 0.5056, \rho_{04}^{(1,2)} = 0.519\,9,$$
$$\rho_{05}^{(1,2)} = 0.4787, \rho_{06}^{(1,2)} = 0.486\,0,$$
$$\rho_{07}^{(1,2)} = 0.5094, \rho_{08}^{(1,2)} = 0.510\,5.$$

Their relation order is

$$X_2 \succ X_1 \succ X_4 \succ X_8 \succ X_7 \succ X_3 \succ X_6 \succ X_5 .$$

It shows that the relationship between the added value of Industry and the GDP is the largest. The smallest is the Farming, Forestry, Fishery, Animal Husbandry and Water Conservancy.

(2) Due to X_2 belonging to the Secondary Industry, X_1 belonging to the Primary Industry. Therefore, "the Secondary Industry" $\succ$ "the Primary Industry" $\succ$ "the Tertiary Industry." These results conform to the actual situation. It indicates that the new model has certain practicality.

(3) We should optimize industry structure. In the aspect of optimizing industrial structure, the Tertiary Industry occupies a leading position in developed country, which is about 65%. This shows that the Tertiary Industry is relatively backward in China. Therefore, we should accelerate the development of the service sector and promote the proportion of the tertiary industry in the GDP.

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3.5 Discussion

There are some limitations in this paper. For example, the quasi-optimal value of *Q-BTGSID* is not necessarily equal to the optimal value, which is a pleased solution. In addition, in the aspect of grey cluster analysis and some quantitative calculation, although their rationality is proved, we do not give some specific examples. All these limitations will be eliminated in future.

4 Conclusions

In this paper, the model of *GAID, GRID* and *GSID* are improved. We propose a new model and proved their rationality, as well. The sensitivity analysis is completed and the quasi-optimal value of *Q-BTGSID* is found based on four principles. We also establish the specific calculation steps of *Q-BTGSID*. To some extent, this will solve the problem as follows:

(1) The quasi-optimal value of *Q-BTGSID* are no longer limited in the range of (0.5, 1], meanwhile, it improves the distinguish effect.

(2) In the aspect of grey cluster analysis, we will choose 0.5 of *r* or some proper number which are less than 0.5, in order to overcome the shortage of original model.

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