## Journal of System Simulation

Volume 27 | Issue 9

Article 10

8-7-2020

# 3D Trajectory Tracking of PX4 Quadrotor Based on Improved Dynamic Inversion

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#### Keywords

quadrotor, trajectory tracking, dynamic inversion, MRAC

### **Recommended Citation**

Hu Yueli, Cai Weiping, Yang Wenrong. 3D Trajectory Tracking of PX4 Quadrotor Based on Improved Dynamic Inversion[J]. Journal of System Simulation, 2015, 27(9): 1997-2007.

第27卷第9期
2015年9月

# **3D** Trajectory Tracking of PX4 Quadrotor Based on Improved Dynamic Inversion

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**Abstract:** An improved dynamic inversion method was proposed for the underactuation and strong coupling in attitude variables of the quadrotors, namely, combining model reference adaptive control (MRAC) with dynamic inversion in the fast variable loop in the system. Through the establishment of dynamic model of the quadrotor and motor model, the control law of the attitude angle and position was deduced. The MATLAB simulation for the scheme was analysed and compared with conventional dynamic inversion, and also simulated in the situation that tracking a given tracjectory with a 10% weight reduction from it's load. Simulation with respect to both scheme show that the improved control structure can quickly adapt to external disturbance and mass change and have higher precision tracking performance than conventional ones.

Keywords: quadrotor; trajectory tracking; dynamic inversion; MRAC

#### 基于改进动态逆的 PX4 四旋翼三维轨迹跟踪

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**摘要:**针对四旋翼飞行器欠驱动与姿态角变量间强耦合的问题提出了一种改进动态逆方法,即在系 统快变量环路中引入模型参考自适应与动态逆相结合的方法。通过建立四旋翼动力学及电机模型推 导出姿态角与位置控制律。对该改进方案进行 MATLAB 仿真分析,与常规动态逆进行比较,并在 负载减少10%情况下跟踪给定轨迹,两种方法仿真实验结果表明,改进后的控制结构能较快适应外 界干扰及负载的变化,具有更精确的路径跟踪性能。

关键词:四旋翼飞行器;轨迹跟踪;动态逆;模型参考自适应

中图分类号: V249.1 文献标识码: A

文章编号: 1004-731X (2015) 09-1997-11

## Introduction

Nowadays, quadrotor UAV is widely used in military, civil and other fields for its simple structure, flexible control, vertical take-off and landing and



Received:2015-05-15 Review: 2015-07-10; Biography: Hu Yueli(1959-), male, Shanghai, China, Ph.D, professor, main research directions: image processing, machine vision, chip system and design, control theory; Cai Weiping(1989-), female, Hubei, China, master graduate student, main research directions: embedded system, research and design of control system; Yang Wenrong (1969-), male, Shanghai, China, Ph.D, associate professor, main research directions: mixed signal integrated circuit design. hover features. A growing interest has been shown in the development of UAVs. In fact, several industries require UAVs to do dangerous, complex, continuous and boring tasks instead of men, such as automotive, medical, manufacturing, space, etc. This attracted widespread interest for researchers, thanks to the great advances in electronics, acrobatic, sensors and control theories, exciting progress in UAVs (Pines and Bohorquez 2006, Salih A L 2010)<sup>[1-2]</sup> have achieved. Extensive results have been presented in

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this area, Sun L (2014) demonstrated a Lyapunov based backstepping trajectory tracking strategy for UAVs with an autopilot in the closed-loop system<sup>[3]</sup>. Sanghyuk Park presented a new guidance logic for trajectory following, and he also using this logic to control two UAVs <sup>[4]</sup>. Daniel Mellinger and his fellow developed an algorithm enables real-time optimal trajectory generation for quadrotors<sup>[5]</sup>.

The control methods commonly used in quadrotor area include: 1)PID and LQ <sup>[6-7]</sup>, 2)backstepping approaches<sup>[8]</sup>, 3)integral sliding mode control approaches <sup>[9]</sup>, 4)model predictive attitude control<sup>[10]</sup> and 5) dynamic inversion based techniques<sup>[11]</sup>. Based on the research of [11], the conventional dynamic inversion methodology will generate dynamic inversion error due to inaccurate modeling, time scale separation and uncertain factors of the UAV. Therefore, it is necessary to take some measures to compensate the inversion error.

This paper mainly makes a contribution of combining MRAC together with dynamic inversion to compensate the inversion error, we focus on the dynamic modeling (section 1) and MRAC dynamic inversion control(section2) for quadrotors, in section2, the pseudo control method is used to adapt to the change of the reference model state. Since the attitude control of quadrotor is of critical importance, with precise attitude control, position control even use PID can achieve great control effect. Here, a trajectory path consists of a set of waypoints defined before the autonomous flight in section 3. In particular, in order to verify the robustness of the improved structure to attitude control, the external disturbance is added to roll and pitch angles during perfect hover, additionally, for path tracking comparison, a 10% weight reduction of it's load was introduced in the simulation experiment. Simulation

results in this section validate the effectiveness of the improved control method, and also indicate that the trajectory tracking of the proposed control system is more accurate and stable than only apply dynamic inversion method in the control of UAV with external disturbance or parameter perturbation.

#### **1** Modeling

#### 1.1 Dynamic Model

Because of the quadrotor aircraft is a complex, multivariable and nonlinear dynamic system, so reasonable assumptions must be made for its modeling. The model built in this paper assumes the following:

1) Aircraft structure is rigid and symmetrical;

2) Propellers symmetrically installed on four-terminal rigid frame and at the same level;

3) The center of the mass is assumed to coincide with the body coordinate system origin.

Before jumping into the mathematic model, some discussion of notation is needed. That's say, the coordinate system, an important aspect of the math model is needed to clarify. The coordinate system will vary on plus ("+") or "x" configuration, this paper uses "x" configuration to model the quadrotor.

As seen in Fig.1, the coordinate system of the x configuration built below consists of the earth fixed coordinate E and body fixed coordinate B. The earth frame,  $ox_E y_E z_E$ , is defined as ENU (with axis  $x_E$ -east direction,  $y_E$  -north direction, and  $z_E$ -upward direction). The body frame we use is defined as having the  $x_B$  axis lie between the arm of rotor 1 (which spins clockwise from above)and 2(spinning in the opposite direction of the adjacent rotors ), with  $y_B$  axis lie between the arm of rotor 2 and 3,

and  $z_B$  axis pointing vertically up during perfect hover. The x axis is assumed to be the positive forward direction for vehicle movement.



Fig. 1 Coordinate systems of x configuration quadrotor

According to the aerospace rotation sequence, the rotation of an aircraft is described as a rotation about  $z_E$  -axis (yaw,  $\psi$ ) then a rotation about the  $x_B$ -axis (pitch,  $\theta$ ) followed by a rotation about the  $y_B$ -axis (roll,  $\phi$ ). Each rotation is made based on a right-handed system and in a single aircraft. Using these three rotations a composite rotation matrix can be created which can transform the motion of the aircraft from the earth frame to a new reference frame. The resulting rotation matrix from the body coordinate with respect to the earth coordinate can be written in the following equation. Below s, c and t represent sine, cosine, and tangent respectively.

$$W_{R_B} = \begin{bmatrix} c_{\theta}c_{\psi} - s_{\phi}s_{\theta}s_{\psi} & -c_{\phi}s_{\psi} & s_{\theta}c_{\psi} + c_{\theta}s_{\phi}s_{\psi} \\ c_{\theta}c_{\psi} + s_{\theta}s_{\phi}c_{\psi} & c_{\phi}c_{\psi} & s_{\theta}s_{\psi} - c_{\theta}s_{\phi}c_{\psi} \\ -s_{\theta}c_{\phi} & s_{\phi} & c_{\theta}c_{\phi} \end{bmatrix}$$
(1)

This rotation matrix is of particular importance in solving the velocity and position state equations. Vector  $[x, y, z]^T$  is the vehicle space location with respect to the earth coordinate. For simplicity, we only consider two main forces acting on the vehicle, gravity (in the  $-z_W$  direction) and thrusts from four rotors,  $T_i$  (in the  $z_B$  direction). The equation of position state shows below describes the linear acceleration of the quadrotor in the earth coordinate

$$m\begin{bmatrix} \ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix} 0\\ 0\\ -mg \end{bmatrix} + W_{R_{B}} \begin{bmatrix} 0\\ 0\\ \\ \\ \sum_{i=1}^{4}T_{i} \end{bmatrix}$$
(2)

Euler angles  $(\phi, \theta, \psi)^T$  are continuous variables of time variation, the derivative of them are not equal to the angular velocity. The components of angular velocity  $(p,q,r)^T$  describes the change rates of the quadrotor in roll, pitch and yaw. The relationship between them can be found through the transform matrix

$$\dot{\Phi} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & t_{\theta} s_{\phi} & t_{\theta} c_{\phi} \\ 0 & c_{\phi} & -s_{\phi} \\ 0 & s_{\phi} / c_{\theta} & c_{\phi} / c_{\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$
(3)

Except for forces, the four rotors can also produces four moments  $M_i$ , perpendicular to the plane of rotation blade.  $I = [I_x, I_y, I_z]^T$  is the three axis moment of inertia of the quadrotor with respect to the body coordinate. The equation of angular acceleration state that taking into account of the three axis torque forces, angular velocity, and inertia across each axis can be written as

$$I\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$
(4)

#### 1.2 Motor Model

The rotation of the four brushless DC motors is the driving force for four rotor propellers of the quadrotor maneuvers, so it is of critical importance to control system design and simulation. The thrust T, provided by a single motor/prop system can be

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calculated as follows:

$$T = C_T \rho A_r r^2 \omega_i^2 \tag{5}$$

Where,  $C_T$  is the thrust coefficient for a specific rotor,  $\rho$  denotes the air density,  $A_r$  denotes the propeller's rotation cross sectional area, r denotes the rotor radius, and  $\omega_i$  denotes the rotor angular velocity. For simplicity, a lumped parameter  $c_T$  can be applied to the characterization process for simple flight modeling:

$$T = c_T \omega_i^2 \tag{6}$$

Similarly, the torque force of the motor or prop system can also be done in a lump parameter  $c_M$ . So the related equation is shown below:

$$M = c_M \omega_i^2 \tag{7}$$

After performing a range of tests with each of the test stands, the provided data analysis programs can help to calculate these coefficients for characterizing the system, see table1. With this information we can create a matrix describing the thrusts and torques on the system like that shown below:

$$\begin{bmatrix} \sum T \\ \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ -Lc_T & Lc_T & Lc_T - Lc_T \\ -Lc_T & -Lc_T & Lc_T & Lc_T \\ -c_M & c_M & -c_M & c_M \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$
(8)

Define L be the arm length from quadrotor hub to motor/prop.

#### 2 Control System Design

In the method of dynamic inversion, the system is divided into fast loop and slow loop according to the response speed of angular velocity, attitude and linear velocity. The slow loop roles as the trajectory tracking loop, mainly control the vertical displacement, vertical and lateral velocity of the vehicle; the fast loop as the attitude tracking loop, controls attitude and angular velocity of the vehicle. When attitude loop start response and stable, tracking loop began, and then gradually stabilize. Therefore, in order to realize the trajectory tracking, attitude control is critical.

#### 2.1 MRAC Dynamic Inversion

As we already known, dynamic inversion is an effective nonlinear direct control method, it avoids a large number of parameter adjustment, but requires a precise mathematical model of the controlled object, so it is often difficult to achieve for complex UAV system. The basic principle of MRAC is that the output response of the designed reference model can express our desired input instructions according to the object structure and control requirements, then the output error between the reference model and the actual system can be used to adjust the controller parameters, so that the actual system can approximately get close to the reference model output.

Consider the following two order system:  

$$\ddot{x} = f(x, \dot{x}, u)$$
(9)

Where  $x \in R^n$  is the state vector,  $u \in R^m$  as control vector, f(x) is a nonlinear dynamic function.

The actual system is very difficult to accurately modeling because of the flight conditions change and uncertain factors, therefore, an approximate function  $\hat{f}(x, \dot{x}, u)$  is usually selected as the approximate system model. Accordingly, the pseudo control variable v is expressed as

$$v = f(x, \dot{x}, u) \tag{10}$$

Applying dynamic inversion method for such system, the actuator commands of dynamic inverse is taken to be of the form

$$u_{cmd} = \hat{f}^{-1} (x_d , \dot{x}_d , v)$$
(11)

Which indicate that with an actuator commands  $u_{cmd}$ , a desired  $\dot{x}_d$  is expected to be

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achieved with great accuracy.

Take the pseudo control into consideration, the reference model will be changed by a new input  $v_h$  if there exist any changes in reference model state. That's, the dynamics modeling of the reference model with pseudo control introduced becomes

$$\ddot{x}_m = f_m \left( x_m, \dot{x}_m, x_d \right) - v_h \tag{12}$$

Where  $x_d$  is the external desired instruction. In this case,  $v_h$  is get based on an approximately estimated control vector  $\hat{u}$ , which is determined by other measurements. Then  $u_{cmd}$  and  $\hat{u}$  are both used to calculate the new input  $v_h$  [12].

$$v_h = \hat{f}\left(x, \dot{x}, u_{cmd}\right) - \hat{f}\left(x, \dot{x}, \hat{u}\right)$$
(13)

 $\dot{x}_d$  has the following form:

$$\dot{x}_d = k \left( x_d - x_m \right) + \mu \left( x_m - x \right) \tag{14}$$

In fact, there always exist model error, under the condition that  $x_m = x_d$ , that is to say, if the reference model following the desired response without static error, then control input is determined

only by the error between reference model and actual system  $\mu(x_m - x)$ . The control structure can be clearly seen in Fig.2.

#### 2.2 Attitude Control

The fast variables of quadrotor vehicle are directly affected by the vehicle modeling error, inverse error and model uncertainties, anyone can imagine if we make the error of the fast variables converge before they influence the remaining variables, then it can overcome disturbances and uncertainties. So, we consider adding an model reference adaptive dynamic inverse controller into the fast variable define loop, and  $\gamma = (\phi, \theta, \psi)^T$ ,  $\omega = (p, q, r)^T$ , angular acceleration be  $\alpha$ , the structure is shown in Fig.3.

Where, dynamic inversion I is obtained from the inverse of eqn(3).



Fig. 2 MRAC dynamic inversion structure



Fig. 3 Attitude loop MRAC dynamic inversion structure

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The reference model is built according to eqn(2) to (4), the related parameter is given in table 1.

Table 1 Model parameters of Quad-rotor				
Parameter	Description	Value		
m	Rigid mass	2.3 kg		
g	Acceleration of gravity	9.81 N/kg		
L	Arm length	0.35 m		
$I_x$	X axis moment of	$9.5 \times 10^{-3} kg m^2$		
	inertia			
$I_y$	Y axis moment of	$9.5 \times 10^{-3} kg \cdot m^2$		
	inertia			
$I_z$	Z axis moment of	$18.6 \times 10^{-3} kg m^2$		
	inertia			
$C_T$	Lift coefficient	$1.48 \times 10^{-7} N$		
		$(rmp^2)^{-1}$		
$c_{M}$	Torque coefficient	2.92×10 <sup>-9</sup> N·		
		$(rmp^2)^{-1}$		

The control law of the fast loop can be achieved by conventional dynamic inverse from the conversion of eqn.(4)

$$u_{1} = \begin{bmatrix} \tau_{\phi d} \\ \tau_{\theta d} \\ \tau_{\psi d} \end{bmatrix} = \begin{bmatrix} qr(I_{z} - I_{y}) + I_{x}\dot{p}_{d} \\ pr(I_{x} - I_{z}) + I_{y}\dot{q}_{d} \\ pq(I_{y} - I_{z}) + I_{z}\dot{r}_{d} \end{bmatrix}$$
(15)

The desired angular acceleration  $\alpha$  can be represented by a first-order inertia as we using the conventional dynamic inverse method.

$$\alpha = \dot{\omega}_d = K_1(\omega_d - \omega_m) \tag{16}$$

Where,  $K_1 = [k_{11}, k_{12}, k_{13}]^T$  is a constant matrix, represent the tracking rate of actual angular velocity output to angular velocity command. After the introduction of MRAC structure, the angular acceleration expression is

$$\alpha' = K_1 (\omega_d - \omega_m) + \mu (\omega_m - \omega)$$
(17)

Where,  $\mu$  is an adaptive element, it can be used to adjust the controller parameters and realize the tracking of reference model. The adaptive element  $\mu$  is constrained by using a steepest descent approach<sup>[13]</sup>, it will stop changing on the condition that the angular velocity of the actual system infinitely get close to angular velocity command.

$$\mu_{n+1} = \mu - 0.0015(\omega_d - \omega) \tag{18}$$

The MRAC dynamic inversion control law was obtained by the fast variable loop according to the above analysis

$$u_{1} = \begin{bmatrix} \tau_{\phi d} \\ \tau_{\theta d} \\ \tau_{\psi d} \end{bmatrix} = \begin{bmatrix} qr(I_{z} - I_{y}) + I_{x}(k_{11}(p_{d} - p_{m}) + \mu(p_{d} - p)) \\ pr(I_{x} - I_{z}) + I_{y}(k_{12}(q_{d} - q_{m}) + \mu(q_{d} - q)) \\ pq(I_{y} - I_{z}) + I_{z}(k_{13}(r_{d} - r_{m}) + \mu(r_{d} - r)) \end{bmatrix}$$
(19)

#### 2.3 3D Trajectory Tracking Control

The position loop equations of the system are given in (2), state variables include the vehicle displacement and linear velocity relative to the world frame, namely,  $W = (x, y, z)^T$ ,  $V_e = (\dot{x}, \dot{y}, \dot{z})^T$ , this set of state variables is required to control the quad-rotor flight trajectory. Slow loop get the attitude and vertical lift control through trajectory instructions  $(x_d, y_d, z_d, \psi_d)$  and the feedback of state variables  $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ . In position loop, the relationship between acceleration  $a = (\ddot{x}, \ddot{y}, \ddot{z})^T$  and  $(T_z, \phi_d, \theta_d)$  is

$$a = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -\frac{T_z}{m} (c_{\phi} s_{\theta} c_{\psi} + s_{\phi} s_{\psi}) \\ -\frac{T_z}{m} (c_{\phi} s_{\theta} s_{\psi} - s_{\phi} c_{\psi}) \\ -\frac{T_z}{m} (c_{\phi} c_{\theta}) + g \end{bmatrix}$$
(20)

Then using small angle approximation principle to linearize (20) to get the relationship between desired roll, pith angles, total thrust and accelerations.

$$\begin{aligned} \ddot{x}_{d} &= -\frac{I_{zd}}{m} \Big( \theta_{d} c_{\psi} + \phi_{d} s_{\psi} \Big) \\ \ddot{y}_{d} &= -\frac{T_{zd}}{m} \Big( \theta_{d} s_{\psi} - \phi_{d} c_{\psi} \Big) \\ \ddot{z}_{d} &= -\frac{T_{zd}}{m} + g \end{aligned}$$
(21)

These formulas are inverted to compute the

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desired roll and pitch euler angles for the attitude control loop, as well as desired thrust  $T_{zd}$ , from the desired accelerations.

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$$\phi_{d} = -\frac{m}{T_{zd}} (\ddot{x}_{d}s_{\psi} - \ddot{y}_{d}c_{\psi})$$
  

$$\theta_{d} = -\frac{m}{T_{zd}} (\ddot{x}_{d}c_{\psi} + \ddot{y}_{d}s_{\psi})$$
  

$$T_{zd} = m(g - \ddot{z}_{d})$$
(22)

PID control method is used to get a,

$$a = k_p e_w + k_i \int (e_w) + k_d e_v \tag{23}$$

Where,  $k_p$ ,  $k_i$ ,  $k_d$  represent the proportional

coefficient, integral coefficient and derivative coefficient respectively,  $e_w = W_d - W$ , is the position vector error,  $e_v = V_d - V_e$ , is the linear velocity vector error.

These together with eqn (19) yield the quadrotor desired body force and moments. From these quantities above, the control inputs for individual quadrotor vehicles are obtained using the control basis vectors.

$$u = \left[T_{zd}, \tau_{\phi d}, \tau_{\phi d}, \tau_{\phi d}\right]^T$$
(24)

Fig.4 shows the position loop control structure.



Fig. 4 Position loop PID dynamic inversion control

## 3 Simulation and Results Analysis

The proposed control architecture was tested in MATLAB /Simulink simulation environment based on the hardware experimental platform built in our laboratory. For the simulation, angles data are provided at 100Hz, position data are provided at 20Hz, max propellers rotation speed is 260 rad/sec, simulation time is 45s.

The simulink based quadrotor simulation here is intended to test the effective of the improved control structure. Fig.5 provides an animated plot that gives a visual representation of the physical responses of the quadrotor from the simulation output data. This GUI was developed to provide a more intuitive view of simulation output than can be offered by simple 2D graphs. Fig.6 also shows a 2D position in X-Y plane. This GUI design displays the quadrotor's attitude relative to the inertial frame, and also displays the quadrotor's position in the inertial frame. The animation is aimed to display the quadrotor in "x" configuration.

Several groups of contrast experiments are carried out in the following part.

Experiment 3.1: Attitude control with disturbance

The goal of this experiment is to stabilize the attitude angles in perfect hover , namely, roll and pitch angles are zero, applying a reverse signal interference d = -8 to roll angle at 30s during the quadrotor hover mode, as well as positive interference signal d = 8, d = 5 respectively applied to pitch angle at 15s and 30s. At the same time, conventional dynamic inversion (DI) and improved dynamic inversion (I-DI) methods are respectively used in the simulation, and the anti-disturbance performance of them are compared in Fig.7.

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Fig. 5 3D attitude and position GUI for PX4 quadrotors



Fig. 6 Position for PX4 quadrotors in X-Y plane GUI



Fig.7 Attitude control comparison of both methods with disturbance

Experimental results in Fig.7 show that with the interference signal applied, although both methods can make attitude angle recover to zero, but the improved dynamic inversion has a smaller overshoot than the traditional ones, anti-disturbance

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Experiment 3.2: Trajectory tracking control without parameter perturbation

performance is stronger than the latter.

Initial condition setting: the PX4 quadrotor UAV takeoff at position (0, 0, 0), carrying 300g load. A task planning trajectory waypoint is shown in table 2. For the control part, two methods discussed in the previous work are used to control the PX4 quadrotor and the tracking effect is observed through 3D graphics.

Table 2 Task planning poir	ıt
----------------------------	----

Planning path	Position(m)			Yaw(rad)
	$x_d$	$y_d$	$Z_d$	d
Home	0	0	0	0
Wp1	0	0	3	0
Wp2	0	3	3	0.5
Wp3	1	3	3	0.5
Wp4	1	2	3	0
Wp5	2	2	3	0
Wp6	2	3	3	1.2
Wp7	3	3	3	1.2
Wp8	3	0	3	0
Land	3	0	0	0

We can see from Fig.8 that the tracking performance of two methods is both satisfactory when there is no parameter perturbation, and also with little difference of tracking performance between these two control structure.

Experiment 3.3: Trajectory tracking with parameter perturbation

Different from experiment 3.2, this experiment takes the parameter perturbation into consideration during the flight. Namely, reduce the load weight of the quadrotor system model by 10% at 5s during the flight, and other flight conditions are the same.

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The simulation results of conventional dynamic inversion and improved dynamic inversion are given in Fig.9. In order to facilitate the comparison, the position and yaw response curves are drawn with respect to both DI and I-DI methods, which can be clearly see in Fig.9(a).







(a) Commands and acutual response with respect to two methods



Fig.9 Comparison between conventional and improved dynamic inversion control of quadrotor with 10% load weight reduced at 5s

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From the simulation results , we can see that both methods have a sudden rise in height when the load is reduced, but the conventional dynamic inversion shows large tracking deviation from the desired trajectory, error in z-axis, nearly 0.5m deviation from commanded altitude, also with continuous fluctuation in x and y axis in the following flight. Fig.9(c) indicate that although with instantaneous height rise, the improved dynamic inversion control can quickly adapt to the change of mass reduction and recover to higher stable, also have precision tracking performance than conventional ones under the same simulation enviroment.

#### 4 Conclusions

The extension of the traditional methods described in this paper aims to work out of the controlled lab environment and make quadrotors useful in many practical scenarios. At the beginning, quadrotors are mainly used in commercial applications for surveillance and aerial videos by simply using GPS to sense the position of the unmanned aerial vehicle. But these vehicles may probably shouldn't fly stable with GPS lost under buildings or with other factors, and obviously the tracking performance is not satisfactory with the requirement of precision path tracking in situations such as soil sampling in agriculture, roadside garbage pickup for environment protection, and so on. The improved controller would make the vehicles more robust to disturbances from wind and also collisions. Based on previous research relating dynamic inversion, a precise mathematical model of the quadrotor is difficult to get, so with model error, perfect tracking cannot be guaranteed. According to this, an improved method was put forward in this paper. The MRAC was added to the fast variable loop in the quadrotor control structure, which has stronger anti-disturbance performance than the conventional method., and it also can get the system quickly adapt to model parameter perturbation. The MATLAB simulation for both two scheme are analyzed based on the reference model parameter selected from the PX4 quadrotor vehicle platform built in our laboratory, with the situation that respectively involved external disturbance and load weight reduction during the flight. The simulation comparison results show that with interference signals applied to roll and pitch angles, the MRAC dynamic inversion control system has a smaller overshoot than the traditional ones, and in the presence of parameter perturbation it also improves tracking performance of the quadrotor, overcomes the inaccuracy modeling and enhances the robustness of the system.

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